

Inferences about Treatment Effects

$$H_0 : \tau_1 = \tau_2 = \dots = \tau_j = 0$$

$$H_1 : \text{Not All } \tau_j = 0$$

One Factor Linear Model (Population Model)

$$Y_{ij} = \mu + \tau_j + \epsilon_{ij}$$

Score on Y for the i th individual in the j th group = Grand Mean + Treatment Effect for Group j + Error

Predicted Score on Y for the i th individual in the j th group

One Factor Linear Model (Prediction for an Individual)

$$\hat{Y}_{ij} = \mu + \tau_j$$

Predicted Score on Y for the i th individual in the j th group = Grand Mean + Treatment Effect for Group j

One Factor Linear Model (Mean for a Group)

$$\mu_{.j} = \mu + \tau_j$$

Group Mean the j th treatment = Grand Mean + Treatment Effect for Group j

One Factor Linear Model
(Population Model)

$$Y_{ij} = \mu + \tau_j + \epsilon_{ij}$$

Score on Y for the i th individual in the j th group = Grand Mean + Treatment Effect for Group j + Error

Predicted Score on Y for the i th individual in the j th group

One Factor Linear Model
(Population Model)

$$Y_{ij} = \mu + \tau_j + \epsilon_{ij}$$

Score on Y for the i th individual in the j th group = Grand Mean + Treatment Effect for Group j + Error

Mean of Group J

One Factor Linear Model
(Population Model)

$$Y_{ij} = \mu_{.j} + \epsilon_{ij}$$

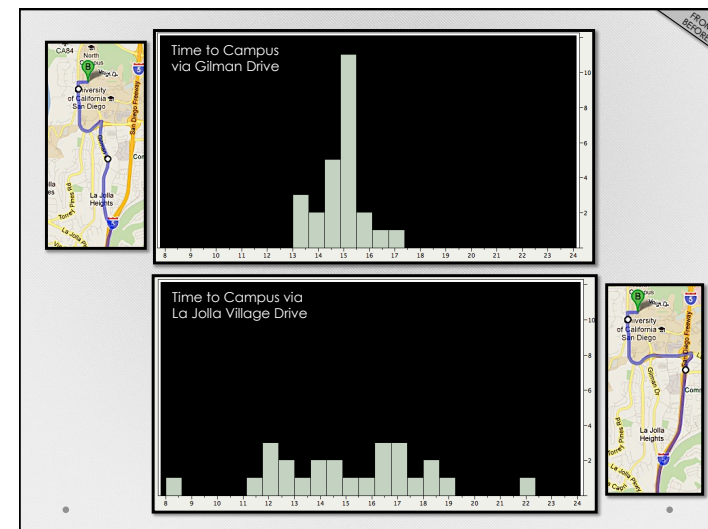
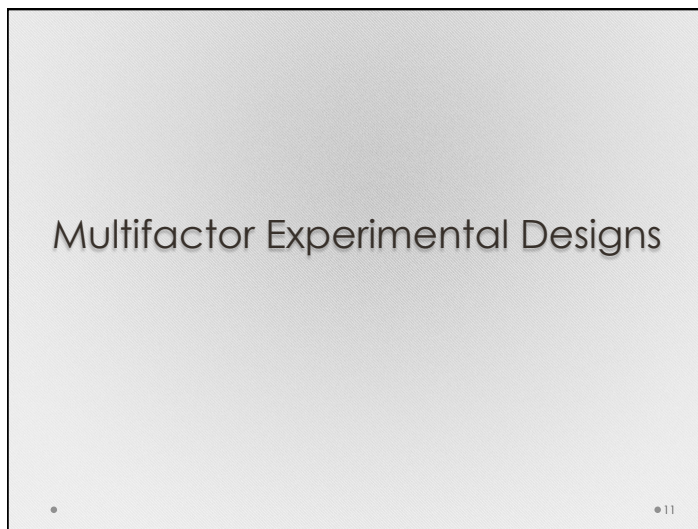
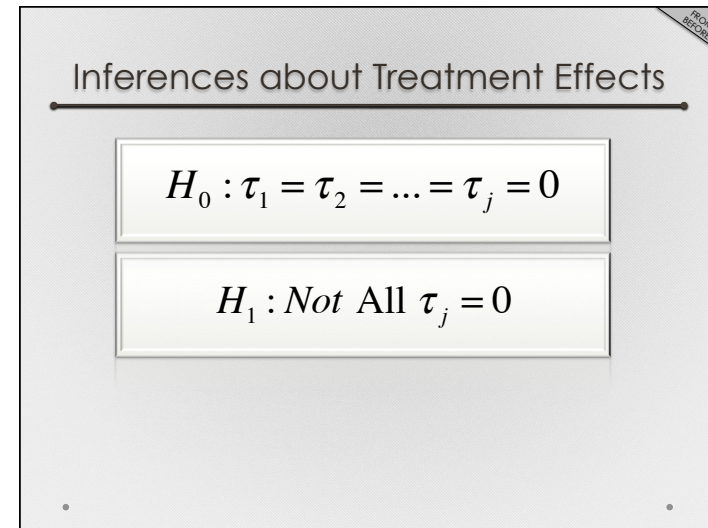
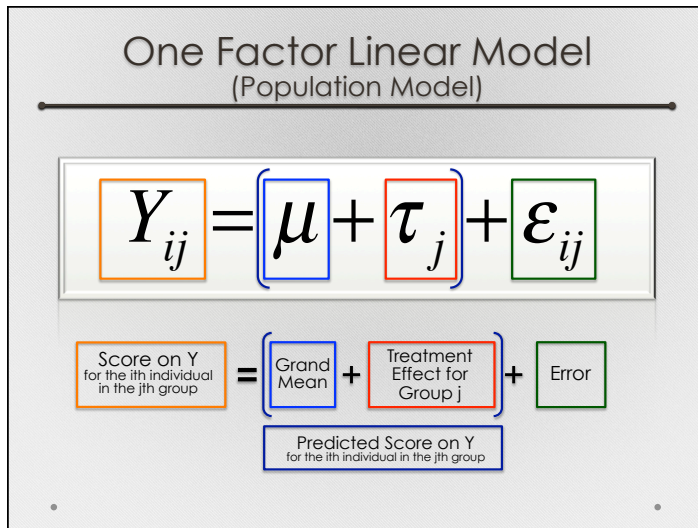
Score on Y for the i th individual in the j th group = Mean of Group j + Error



One Factor Linear Model
(Population Model)

$$Y_{ij} = \mu + \tau_j + \epsilon_{ij}$$

Score on Y for the i th individual in the j th group = Grand Mean + Treatment Effect for Group j + Error

Mean of Group J



| | | |
|---|-------------------------------|--|
|  | Gilman Drive | Average Time Taking Gilman Drive |
|  | La Jolla Village Drive | Average Time Taking La Jolla Village Drive |


Possible Test:
- Effect of Route

● 13

| 8:00am | 9:30am |
|----------------------------------|----------------------------------|
| Average Time to Campus at 8:00am | Average Time to Campus at 9:30am |

Possible Tests:
- Effect of Time

● 14

| | | Factor B | |
|---|-------------------------------|---|---|
| | | 8:00am | 9:30am |
| Factor A  | Gilman Drive | Average Time Taking Gilman Drive at 8:00am | Average Time Taking Gilman Drive at 9:30am |
| | La Jolla Village Drive | Average Time Taking La Jolla Village Drive at 8:00 am | Average Time Taking La Jolla Village Drive at 9:30 am |

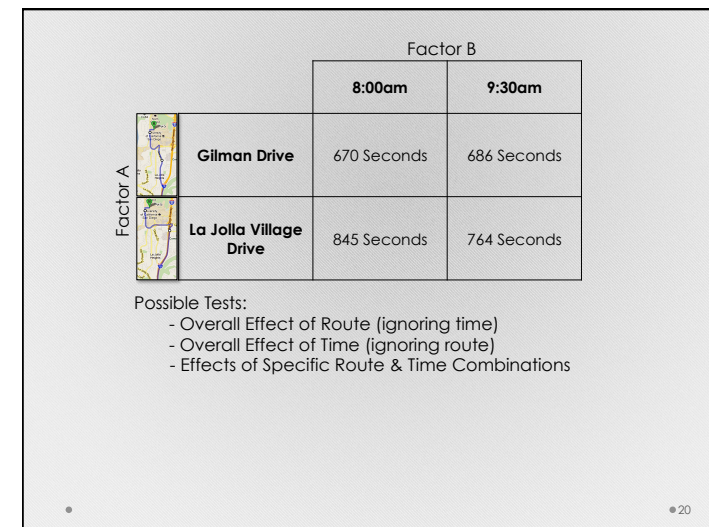
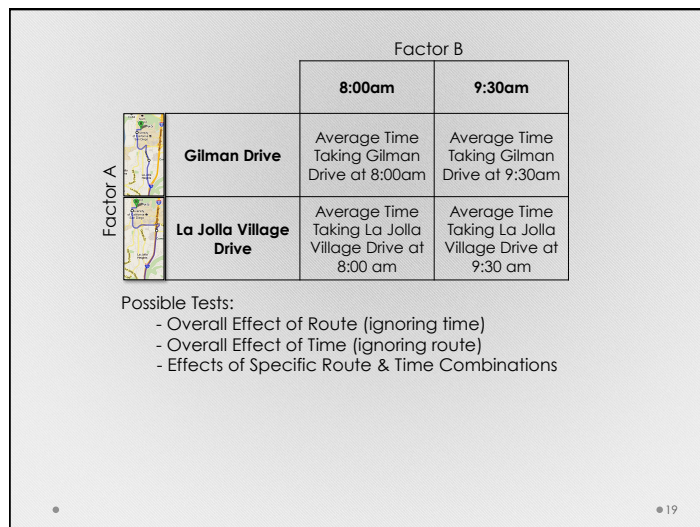
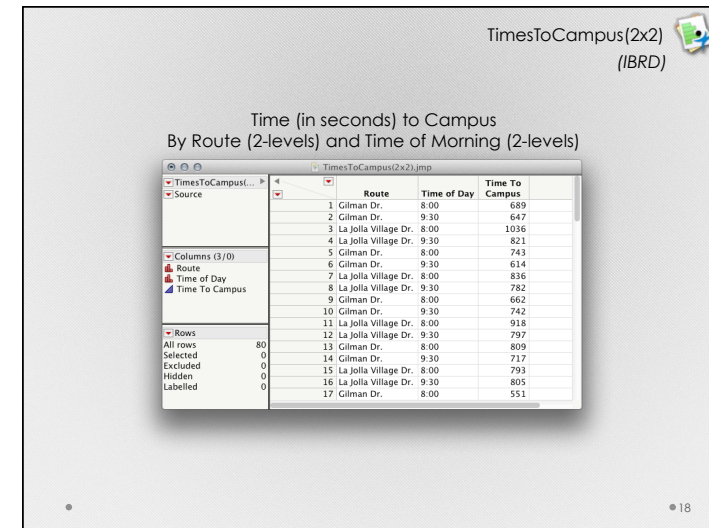
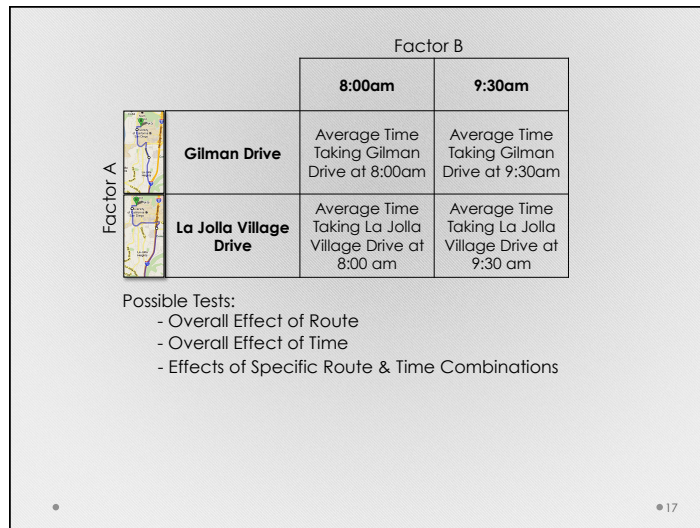
Possible Tests:
- Overall Effect of Route
- Overall Effect of Time
- Effects of Specific Route & Time Combinations

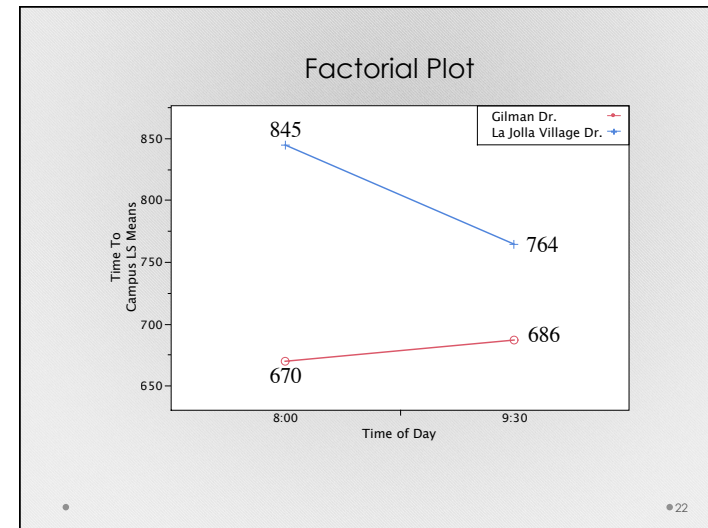
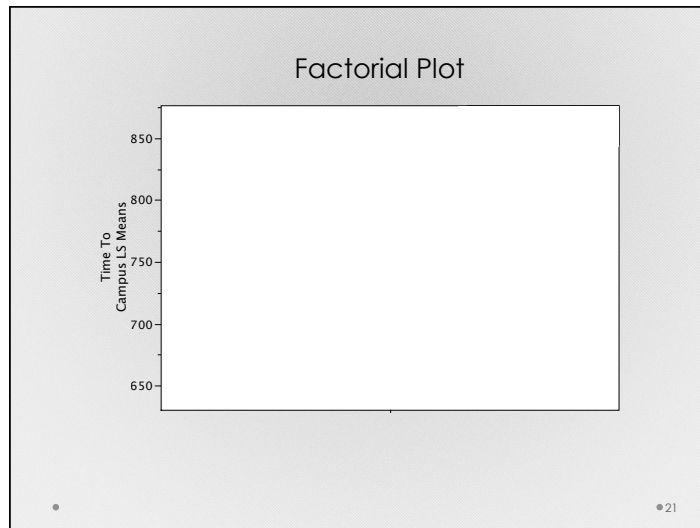
● 15

Factorial Design

Factorial Designs are multifactor designs in which two or more factors are completely crossed; measurements are taken for every combination of factor levels

● 16





| | | Factor B | |
|----------|------------------------|-------------|-------------|
| | | 8:00am | 9:30am |
| Factor A | Gilman Drive | 670 Seconds | 686 Seconds |
| | La Jolla Village Drive | 845 Seconds | 764 Seconds |

Possible Tests:

- Overall Effect of Route (ignoring time)
- Overall Effect of Time (ignoring route)
- Effects of Specific Route & Time Combinations

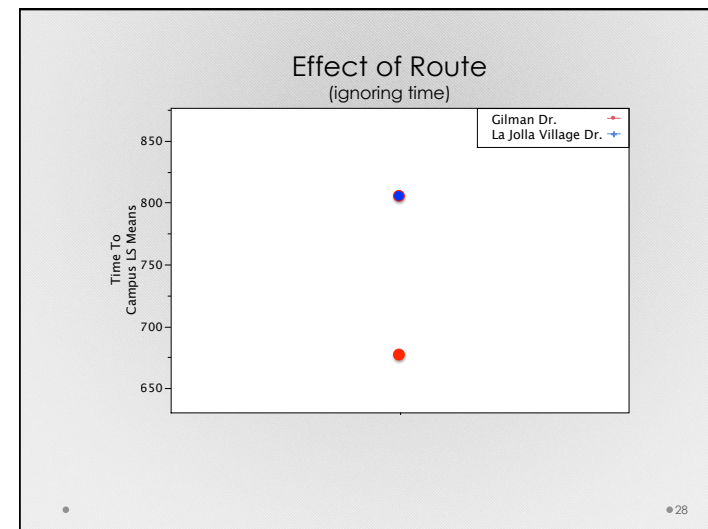
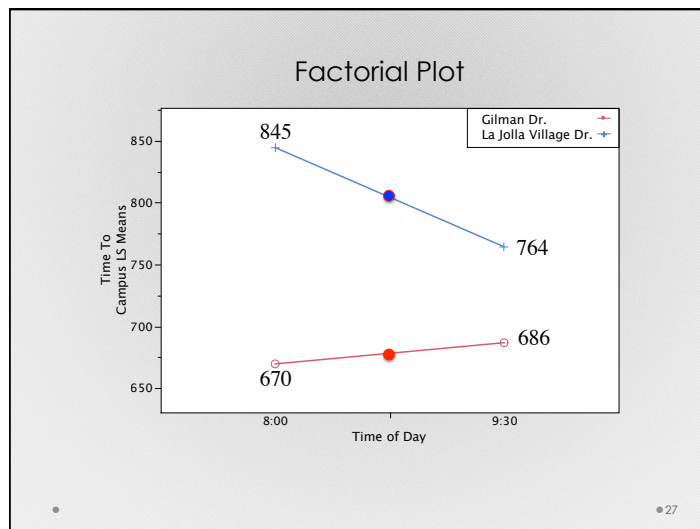
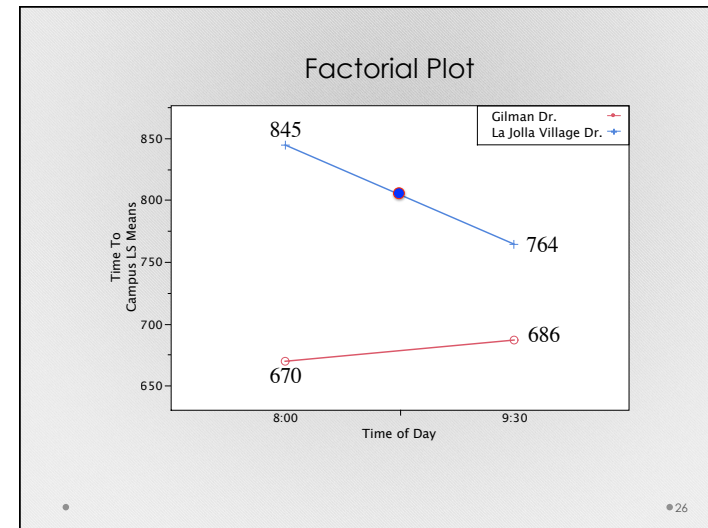
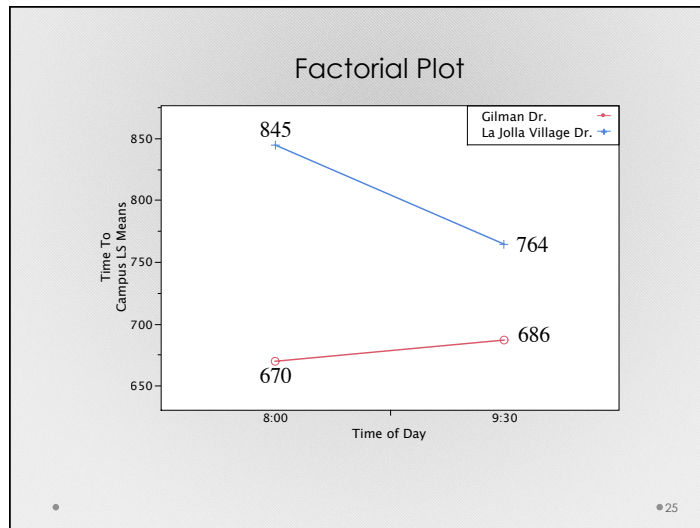
● 23

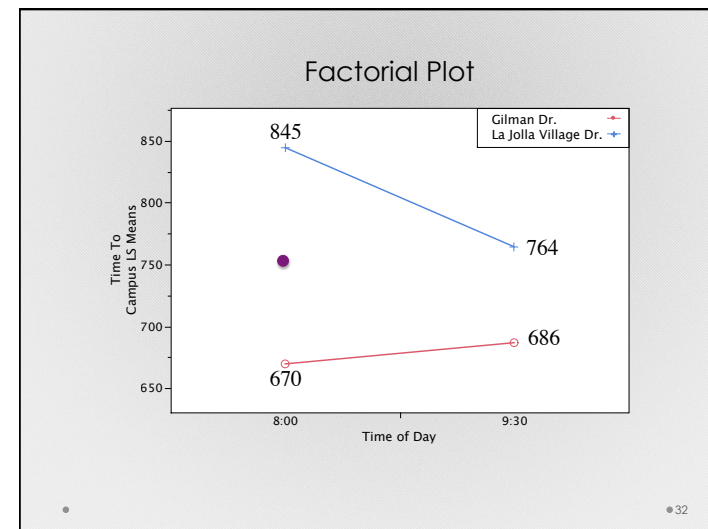
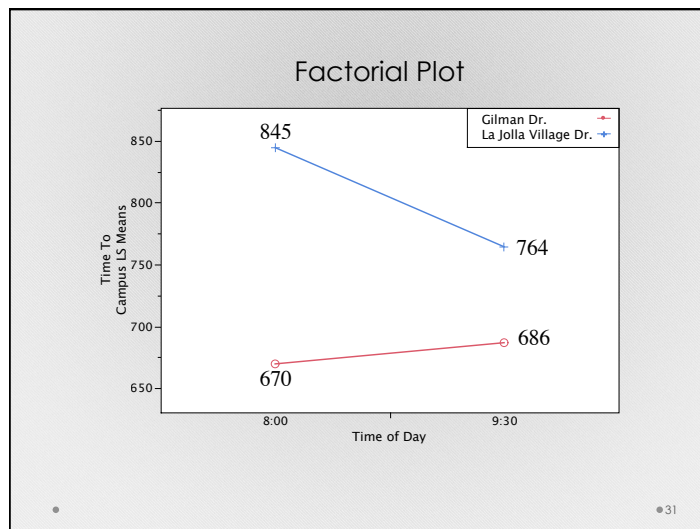
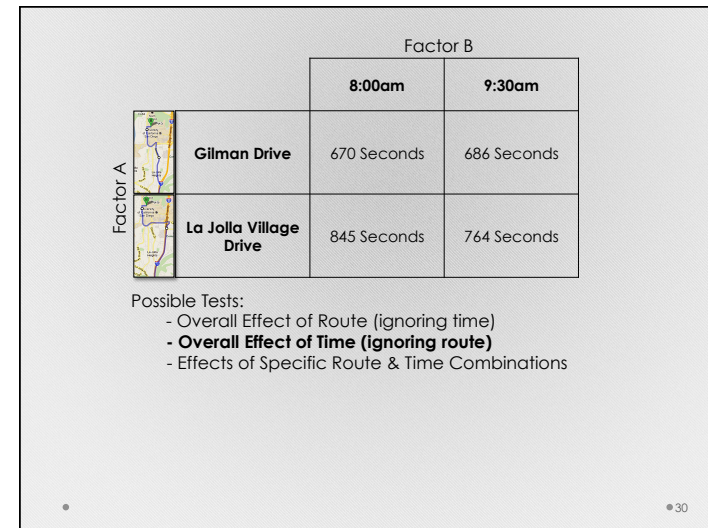
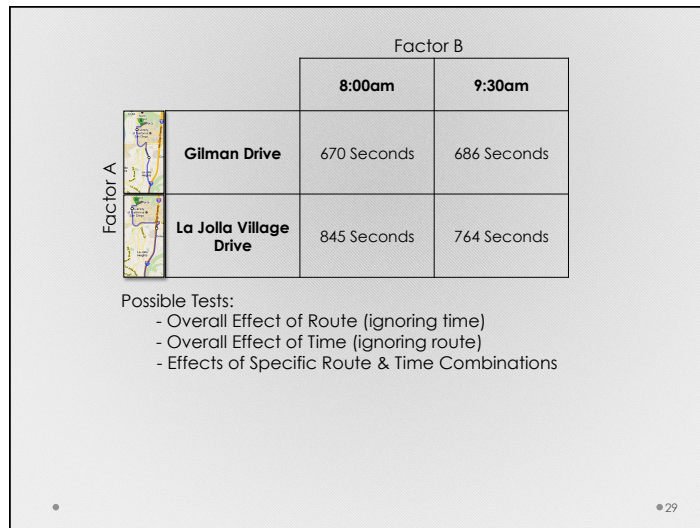
| | | Factor B | |
|----------|------------------------|-------------|-------------|
| | | 8:00am | 9:30am |
| Factor A | Gilman Drive | 670 Seconds | 686 Seconds |
| | La Jolla Village Drive | 845 Seconds | 764 Seconds |

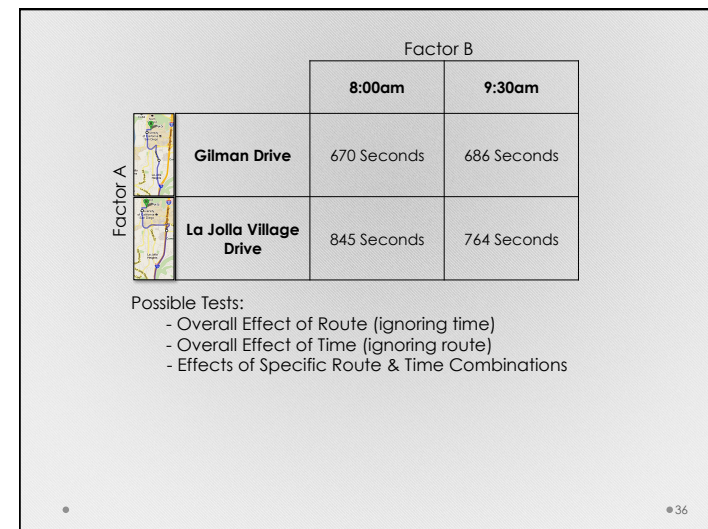
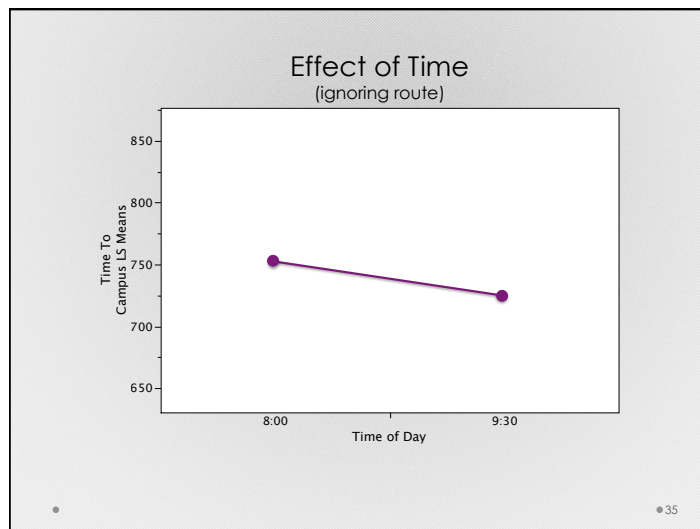
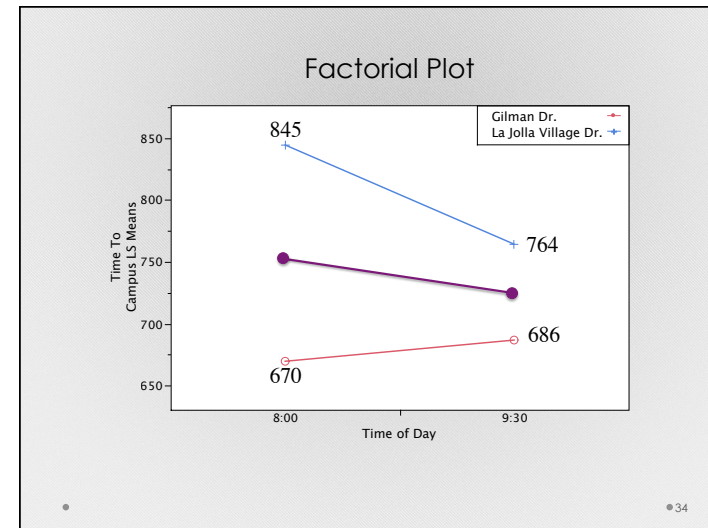
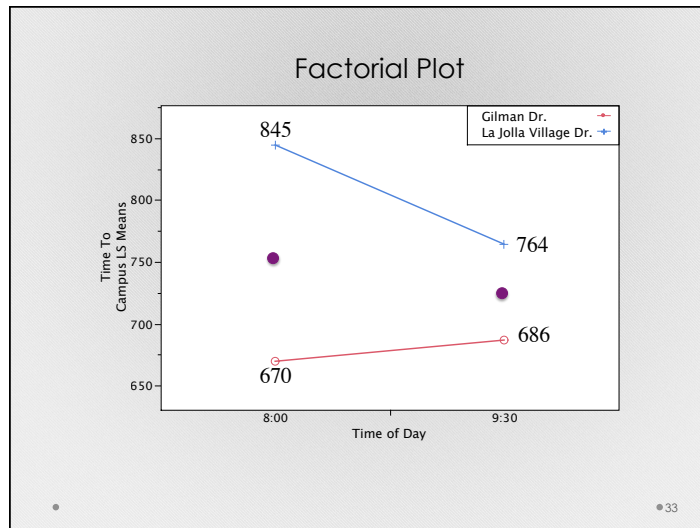
Possible Tests:

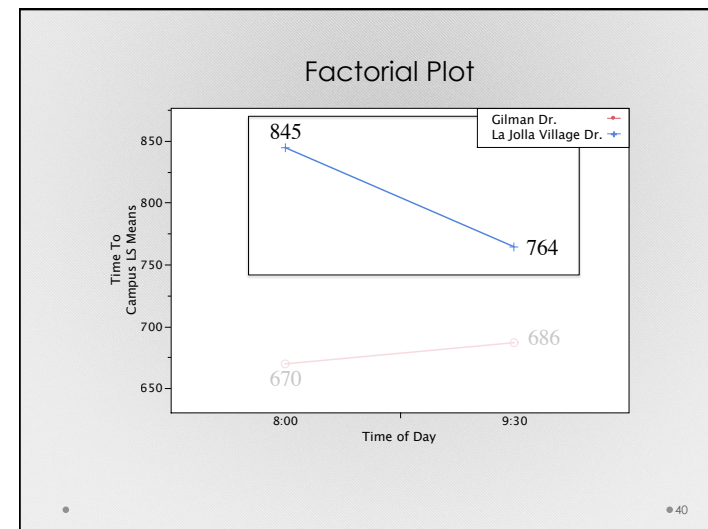
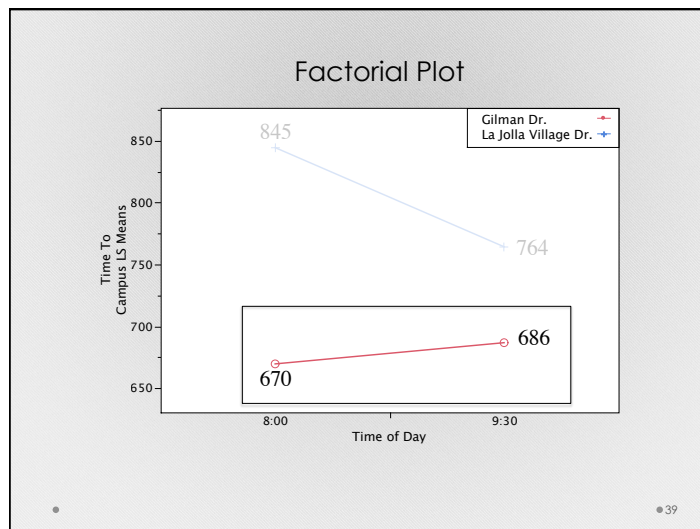
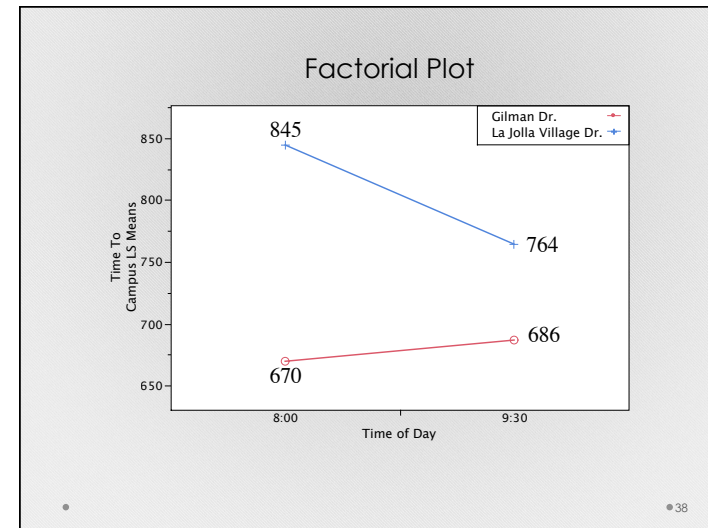
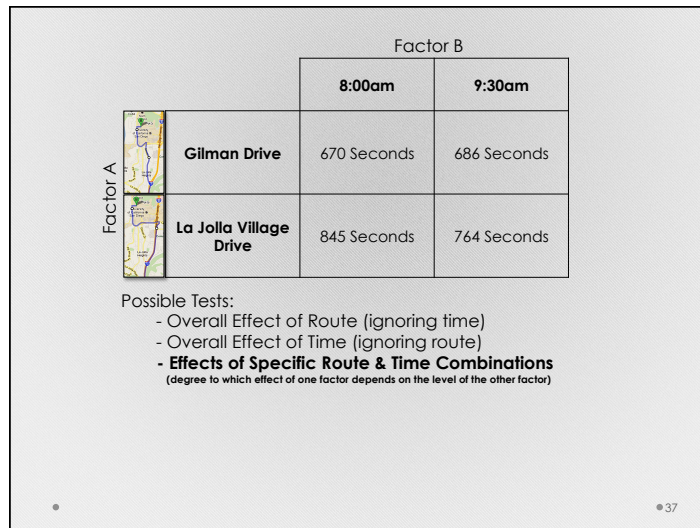
- **Overall Effect of Route (ignoring time)**
- Overall Effect of Time (ignoring route)
- Effects of Specific Route & Time Combinations

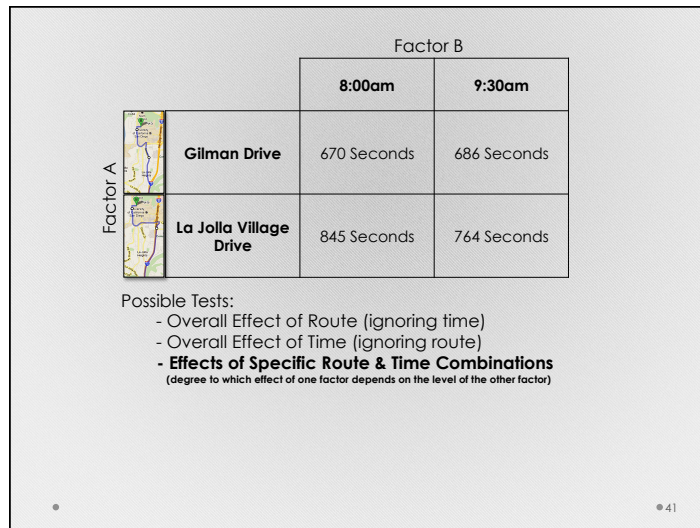
● 24







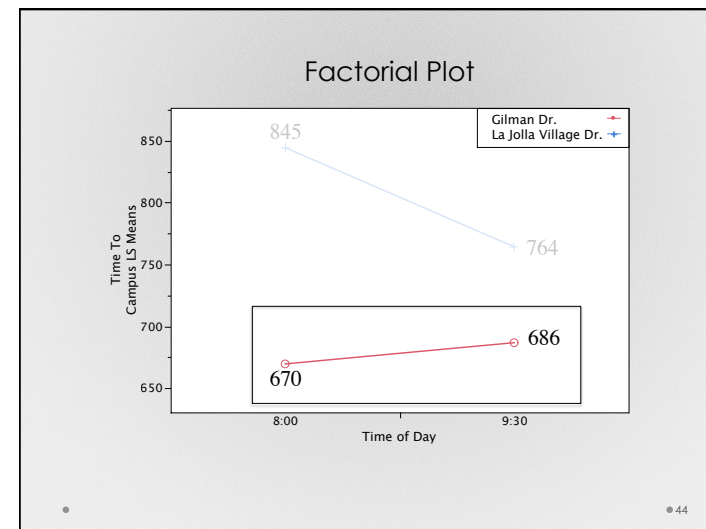
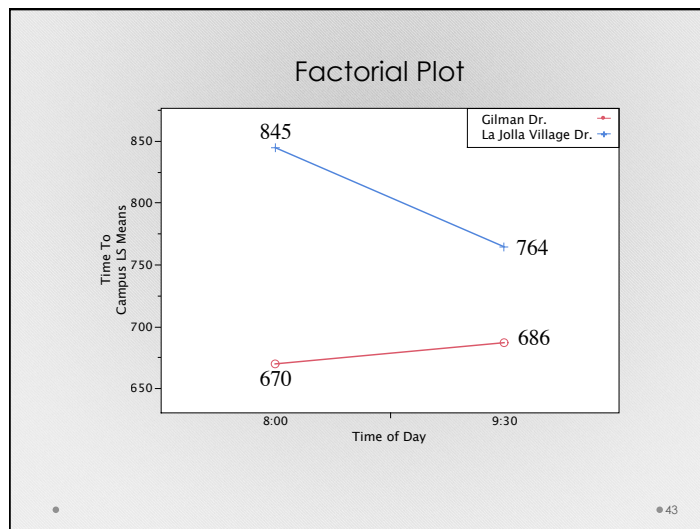


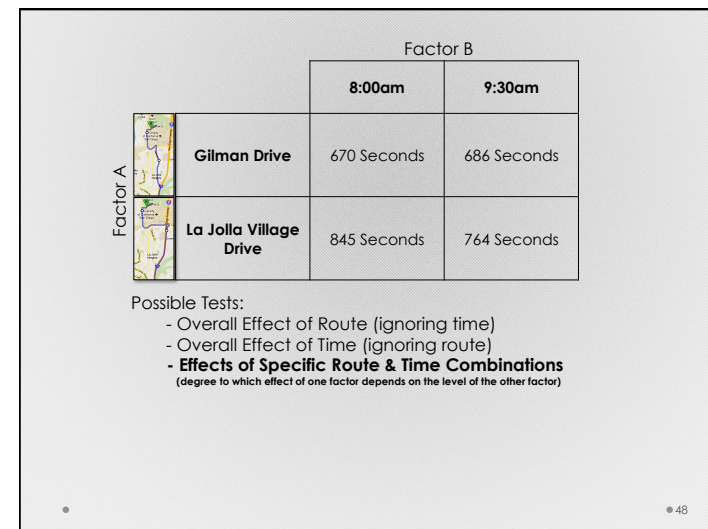
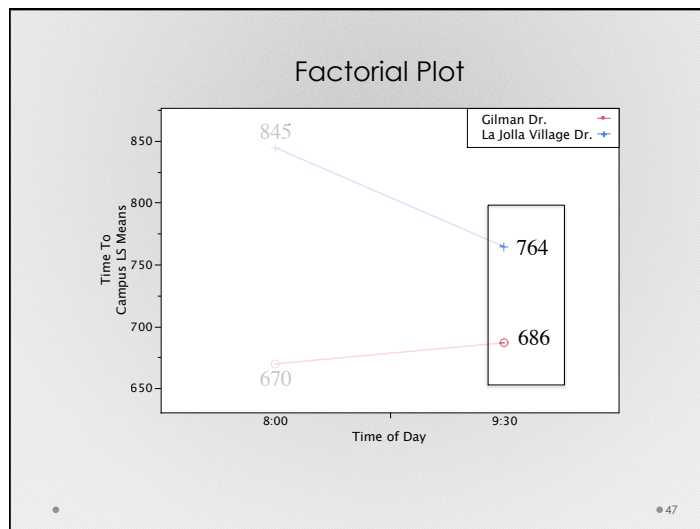
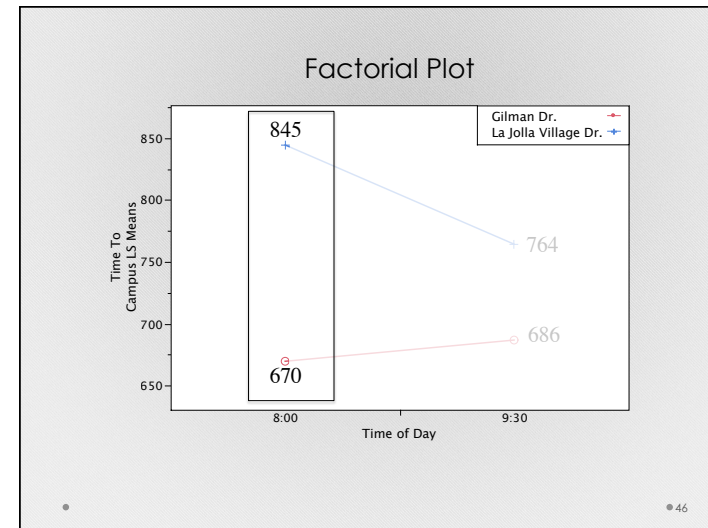
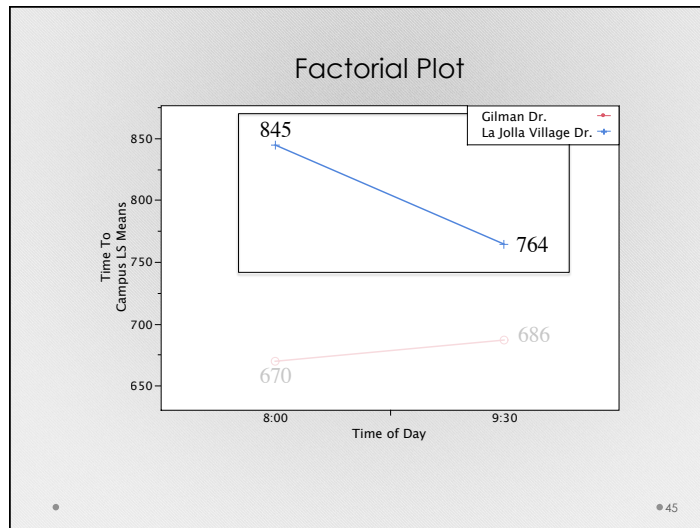




Interaction

The interaction between factors is the degree to which the effects of one factor depend on the level of the other factor

• 42





| | | Factor B | |
|----------|--|-------------|-------------|
| | | 8:00am | 9:30am |
| Factor A |  Gilman Drive | 670 Seconds | 686 Seconds |
| |  La Jolla Village Drive | 845 Seconds | 764 Seconds |

Possible Tests:

- Overall Effect of Route (ignoring time)
- Overall Effect of Time (ignoring route)
- Effects of Specific Route & Time Combinations
(degree to which effect of one factor depends on the level of the other factor)

• 49

One Factor Linear Model (Population Model)

$$Y_{ij} = \mu + \tau_j + \epsilon_{ij}$$

Score on Y for the i th individual in the j th group = Grand Mean + Treatment Effect for Group j + Error

• 50

One Factor Linear Model (Population Model)

$$Y_{ij} = \left[\mu + \tau_j \right] + \epsilon_{ij}$$

Score on Y for the i th individual in the j th group = Grand Mean + Treatment Effect for Group j + Error

Mean of Group J

• 51

One Factor Linear Model (Population Model)

$$Y_{ij} = \mu + \tau_j + \epsilon_{ij}$$

Score on Y for the i th individual in the j th group = Grand Mean + Treatment Effect for Group j + Error

• 52

Two Factor Linear Model (Population Model)

$$Y_{ijk} =$$

Score on Y
for the *i*th
individual in
the *j*th
treatment of
factor A, and
*k*th treatment
of factor B

=

• 53

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu$$

Score on Y
for the *i*th
individual in
the *j*th
treatment of
factor A, and
*k*th treatment
of factor B

=

Grand
Mean

• 54

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j$$

Score on Y
for the *i*th
individual in
the *j*th
treatment of
factor A, and
*k*th treatment
of factor B

=

Grand
Mean

+

Effect
offset for
level *j* of
Factor A

• 55

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j + \beta_k$$

Score on Y
for the *i*th
individual in
the *j*th
treatment of
factor A, and
*k*th treatment
of factor B

=

Grand
Mean

+

Effect
offset for
level *j* of
Factor A

+

Effect
offset for
level *k* of
Factor B

• 56

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk*

• 57

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

• 58

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

• 59

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j + \beta_k + \gamma_{jk} + \epsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

• 60

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \varepsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

• 61

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \varepsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

• 62

One Factor Linear Model (Population Model)

$$Y_{ij} = \mu + \tau_j + \varepsilon_{ij}$$

Score on Y for the *i*th individual in the *j*th group = Grand Mean + Treatment Effect for Group *j* + Error

Predicted Score on Y for the *i*th individual in the *j*th group

FROM SCORE

•

One Factor Linear Model (Population Model)

$$Y_{ij} = \mu + \tau_j + \varepsilon_{ij}$$

Score on Y for the *i*th individual in the *j*th group = Grand Mean + Treatment Effect for Group *j* + Error

Mean of Group *J*

FROM SCORE

•

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \varepsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B

= Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

Predicted Score on Y for the *i*th individual in the *j*th level of Factor A and *k*th level of Factor B

65

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \varepsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B

= Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

Mean of Group for the *j*th level of Factor A and *k*th level of Factor B

66

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu_{\cdot jk} + \varepsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B

= Mean of Group JK + Error

67

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \varepsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B

= Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

Mean of Group for the *j*th level of Factor A and *k*th level of Factor B

68

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu_{...} + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

• 69

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu_{...} + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

• 70

Two Factor Linear Model (Sample Model)

$$Y_{ijk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk} + e_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk* + Error

Mean of Group for the *j*th level of Factor A and *k*th level of Factor B

• 71

Two Factor Linear Model (Sample Model)

$$Y_{ijk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Effect offset for unique effect of Factors in treatment *jk*

Mean of Group for the *j*th level of Factor A and *k*th level of Factor B

• 72

Two Factor Linear Model (Sample Model for Treatment Mean)

$$\bar{Y}_{\cdot jk} = \bar{Y}_{\dots} + a_j + b_k + (ab)_{jk}$$

Mean of Y for the jth treatment of factor A, and kth treatment of factor B = Grand Mean + Effect offset for level j of Factor A + Effect offset for level k of Factor B + Effect offset for unique effect of Factors in treatment jk

• 73

| | | Factor B | |
|----------|------------------------|---|---|
| | | 8:00am | 9:30am |
| Factor A | Gilman Drive | Average Time Taking Gilman Drive at 8:00am | Average Time Taking Gilman Drive at 9:30am |
| | La Jolla Village Drive | Average Time Taking La Jolla Village Drive at 8:00 am | Average Time Taking La Jolla Village Drive at 9:30 am |

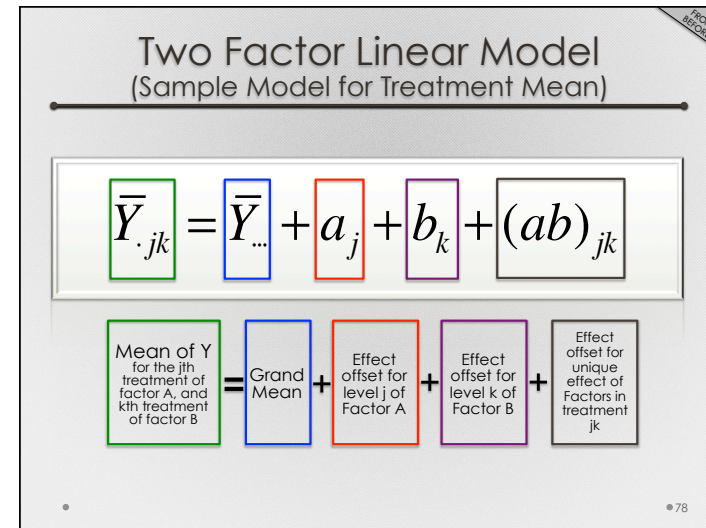
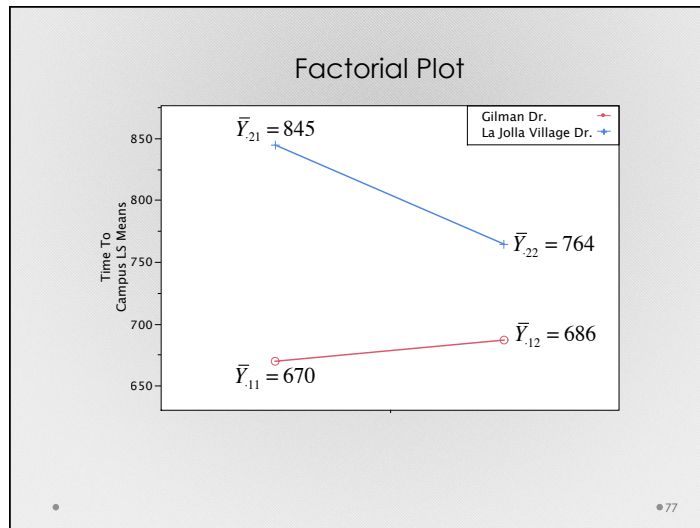
• 74

| | | Factor B | |
|----------|---------------------------------|----------------------|----------------------|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{\cdot 11}$ | $\bar{Y}_{\cdot 12}$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{\cdot 21}$ | $\bar{Y}_{\cdot 22}$ |

• 75

| | | Factor B | |
|----------|---------------------------------|----------------------------|----------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{\cdot 11} = 670$ | $\bar{Y}_{\cdot 12} = 686$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{\cdot 21} = 845$ | $\bar{Y}_{\cdot 22} = 764$ |

• 76



$$\bar{Y}_{jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | |
|----------|---------------------------------|----------------------|----------------------|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{11} = 670$ | $\bar{Y}_{12} = 686$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{21} = 845$ | $\bar{Y}_{22} = 764$ |

• 79

$$\bar{Y}_{jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | |
|----------|---------------------------------|---|---|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{...} + a_1 + b_1 + (ab)_{11}$ | $\bar{Y}_{...} + a_1 + b_2 + (ab)_{12}$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{...} + a_2 + b_1 + (ab)_{21}$ | $\bar{Y}_{...} + a_2 + b_2 + (ab)_{22}$ |

• 80

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | |
|----------|------------------------------------|-----------------|-----------------|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | Gilman Drive j = 1 | | |
| | La Jolla Village Drive j = 2 | | |

•

• 81

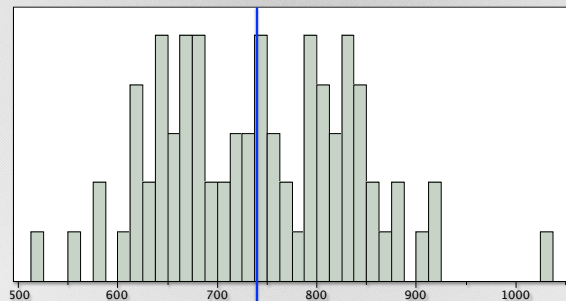
$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | |
|----------|------------------------------------|-----------------|-----------------|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | Gilman Drive j = 1 | | |
| | La Jolla Village Drive j = 2 | | |

•

• 82

Times to Campus



$$\bar{Y}_{...} = 741$$

•

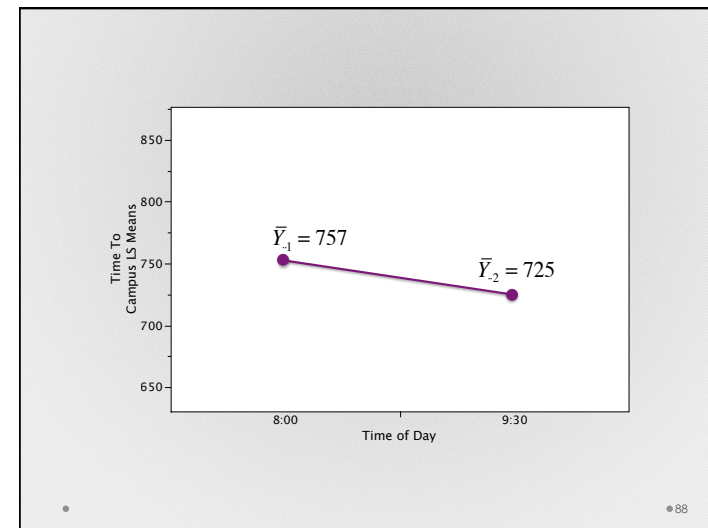
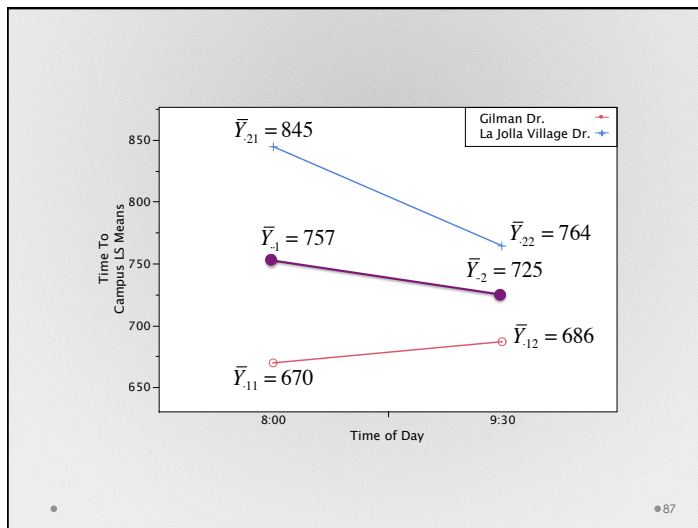
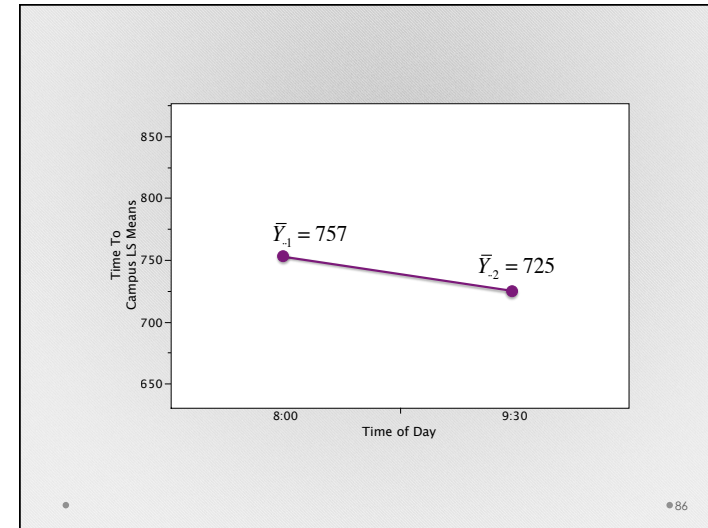
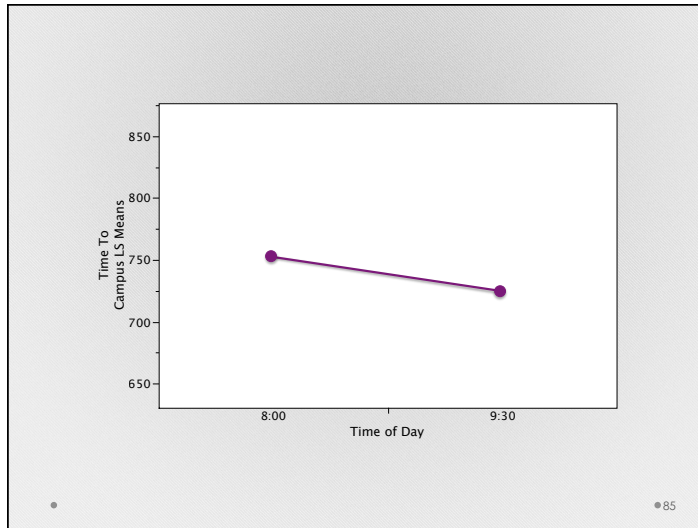
• 83

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | |
|----------|------------------------------------|------------------|------------------|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | Gilman Drive j = 1 | | |
| | La Jolla Village Drive j = 2 | | |
| | | $\bar{Y}_{.1} =$ | $\bar{Y}_{.2} =$ |

•

• 84



$$\bar{Y}_{jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | |
|----------|------------------------------------|-----------------------|-----------------------|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | $\bar{Y}_{...} = 741$ | | |
| | Gilman Drive j = 1 | | |
| | La Jolla Village Drive j = 2 | | |
| | | $\bar{Y}_{\cdot 1} =$ | $\bar{Y}_{\cdot 2} =$ |

• 89

$$\bar{Y}_{jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | |
|----------|------------------------------------|---------------------------|---------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | $\bar{Y}_{...} = 741$ | | |
| | Gilman Drive j = 1 | | |
| | La Jolla Village Drive j = 2 | | |
| | | $\bar{Y}_{\cdot 1} = 757$ | $\bar{Y}_{\cdot 2} = 725$ |

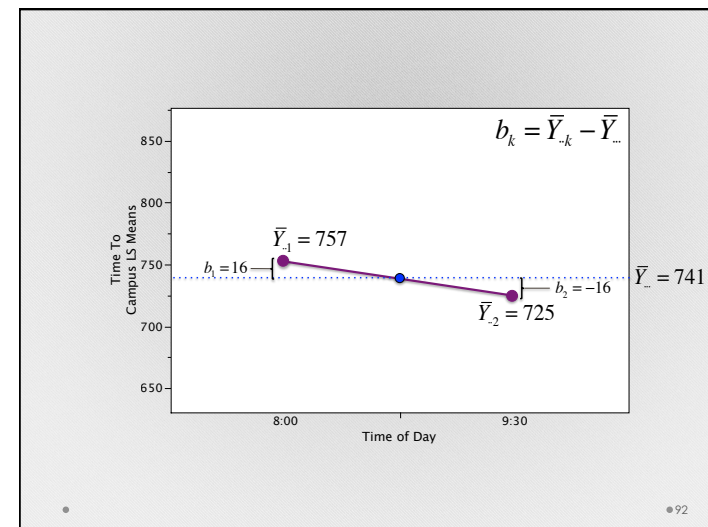
• 90

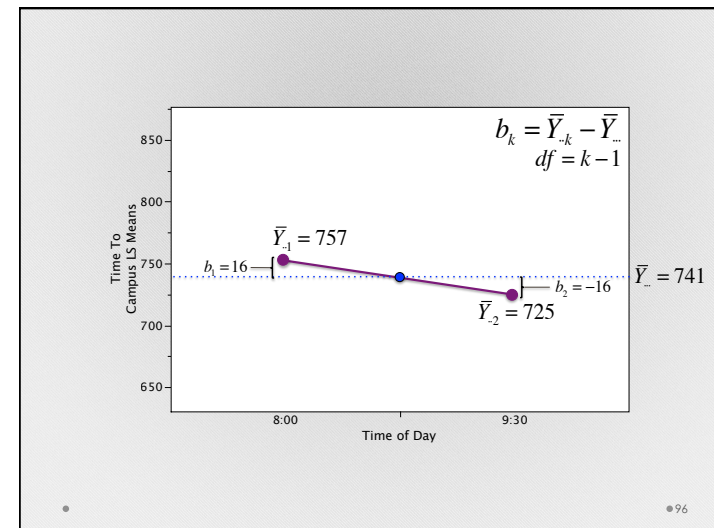
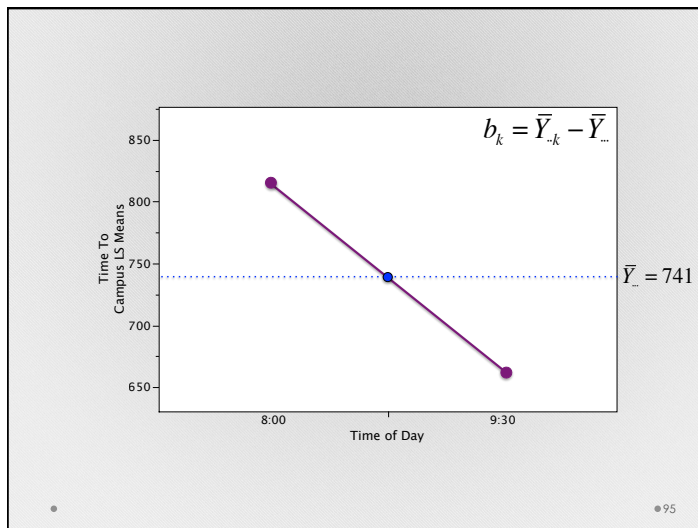
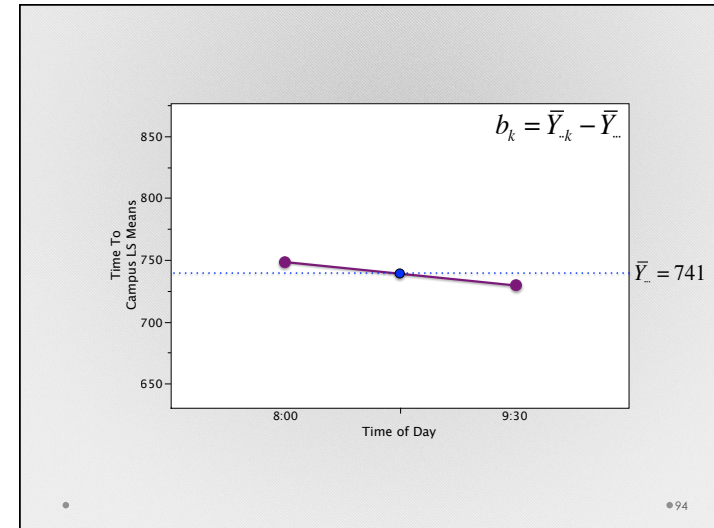
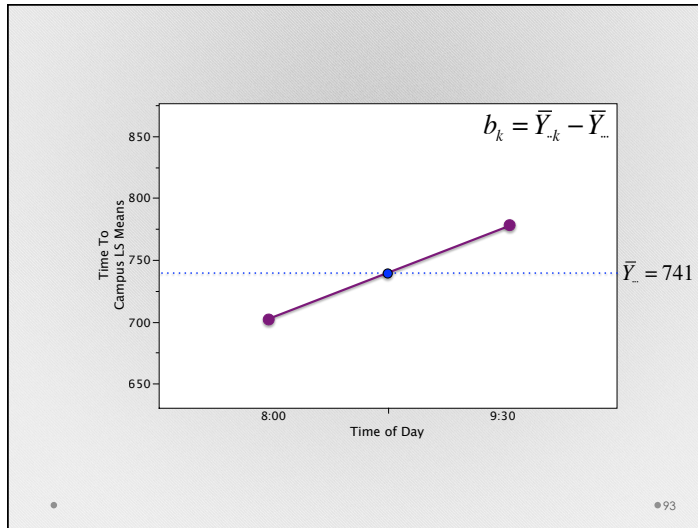
$$\bar{Y}_{jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | |
|----------|------------------------------------|---|--|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | $\bar{Y}_{...} = 741$ | | |
| | Gilman Drive j = 1 | $b_1 = 16$ | $b_2 = -16$ |
| | La Jolla Village Drive j = 2 | $b_1 = 16$ | $b_2 = -16$ |
| | | $\bar{Y}_{\cdot 1} = 757$ $b_1 = 16$ | $\bar{Y}_{\cdot 2} = 725$ $b_2 = -16$ |

$b_k = \bar{Y}_{\cdot k} - \bar{Y}_{...}$

• 91





$$\bar{Y}_{\cdot jk} = \bar{Y}_{\dots} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | | $b_k = \bar{Y}_{\cdot \cdot k} - \bar{Y}_{\dots}$ |
|----------|---------------------------------|---|--|---|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | $\bar{Y}_{\dots} = 741$ | | | |
| | Gilman Drive j = 1 | $b_1 = 16$ | $b_2 = -16$ | |
| | La Jolla Village Drive j = 2 | $b_1 = 16$ | $b_2 = -16$ | |
| | | $\bar{Y}_{\cdot 1} = 757$ $b_1 = 16$ | $\bar{Y}_{\cdot 2} = 725$ $b_2 = -16$ | |

• 97

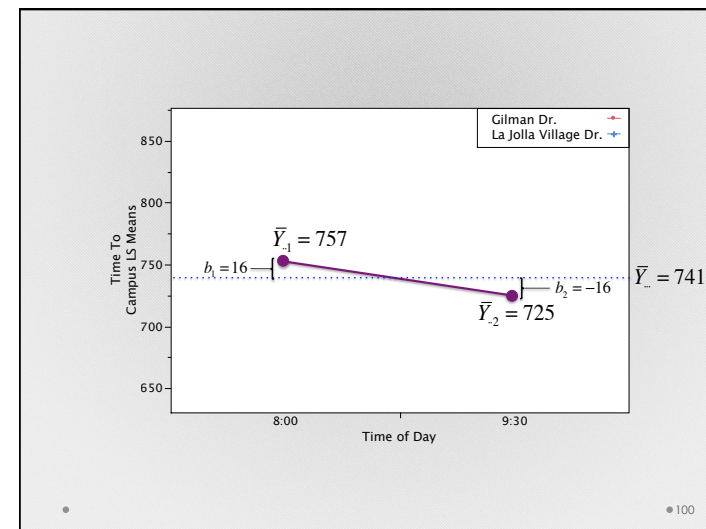
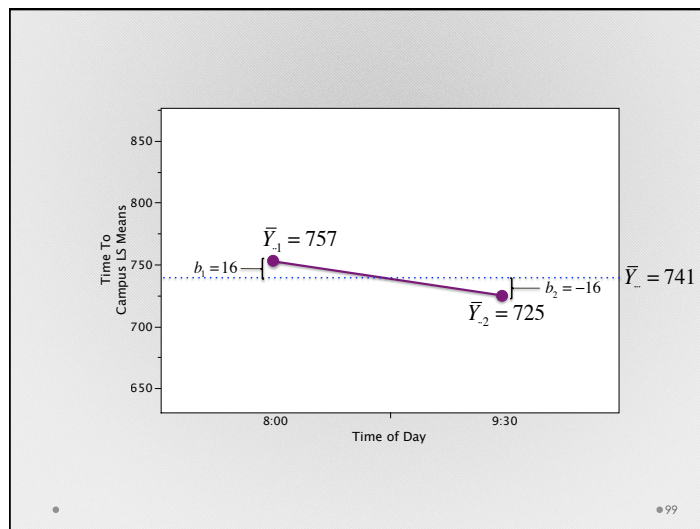
$$\bar{Y}_{jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

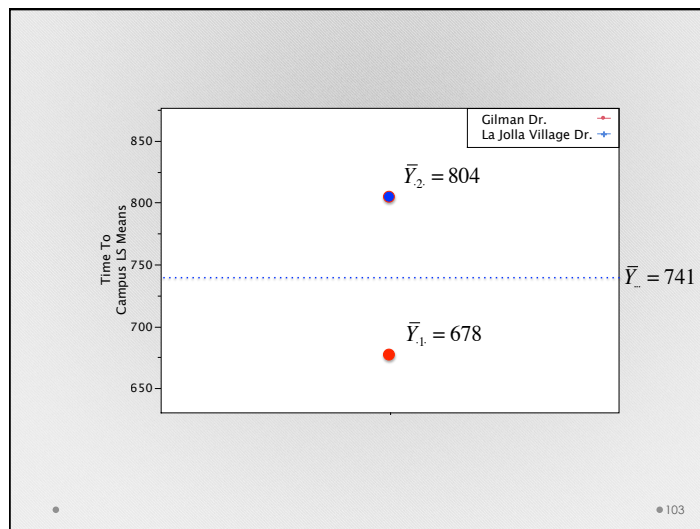
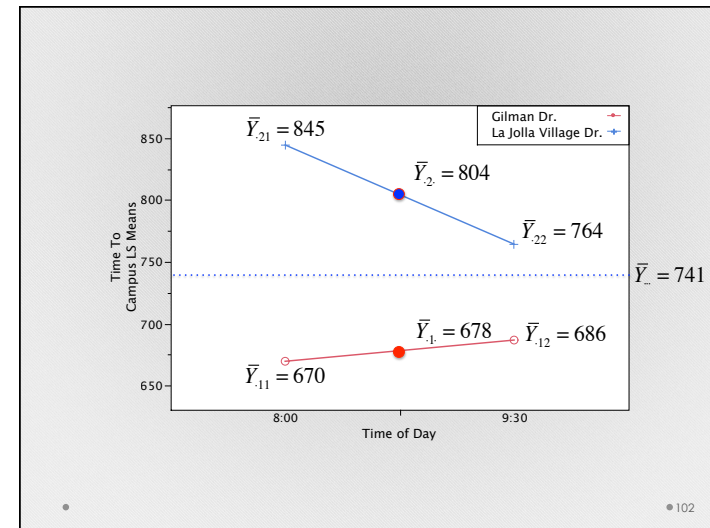
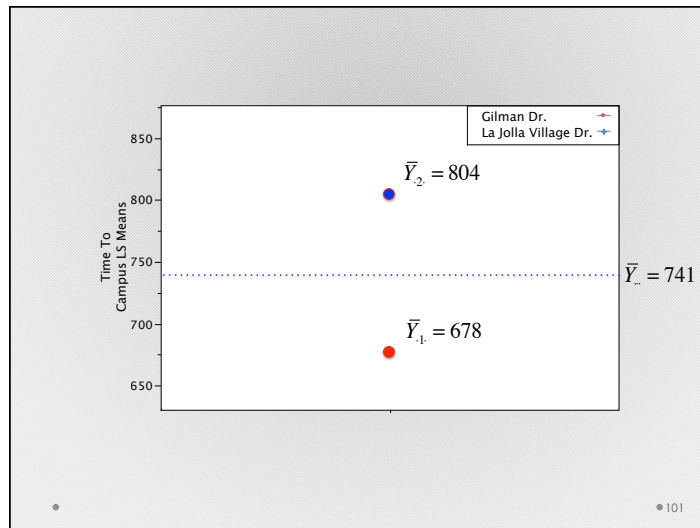
| | | Factor B | | |
|----------|---------------------------------|-----------------|-----------------|------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $b_1 = 16$ | $b_2 = -16$ | $\bar{Y}_{.1} =$ |
| | La Jolla Village Drive j = 2 | $b_1 = 16$ | $b_2 = -16$ | $\bar{Y}_{.2} =$ |

$\bar{Y}_{...} = 741$

$\bar{Y}_{.1} = 757$
 $b_1 = 16$

$\bar{Y}_{.2} = 725$
 $b_2 = -16$





$$\bar{Y}_{.jk} = \bar{Y}_{..} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | | |
|----------|------------------------------------|------------------------------------|-------------------------------------|------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $b_1 = 16$ | $b_2 = -16$ | $\bar{Y}_{.1} =$ |
| | La Jolla Village Drive j = 2 | $b_1 = 16$ | $b_2 = -16$ | $\bar{Y}_{.2} =$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

$$\bar{Y}_{jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

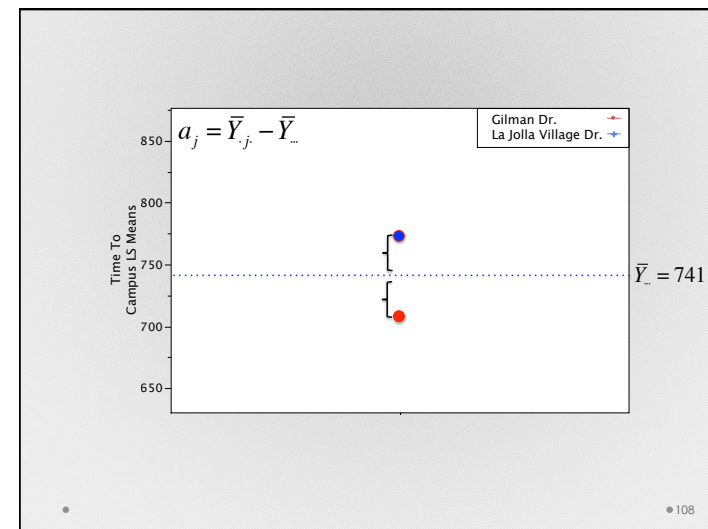
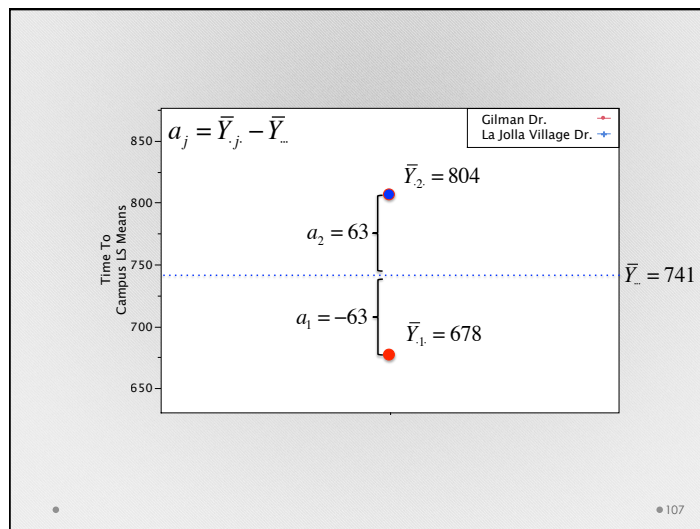
| | | Factor B | | |
|----------|---------------------------------|------------------------------------|-------------------------------------|----------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $b_1 = 16$ | $b_2 = -16$ | $\bar{Y}_{.1} = 678$ |
| | La Jolla Village Drive j = 2 | $b_1 = 16$ | $b_2 = -16$ | $\bar{Y}_{.2} = 804$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

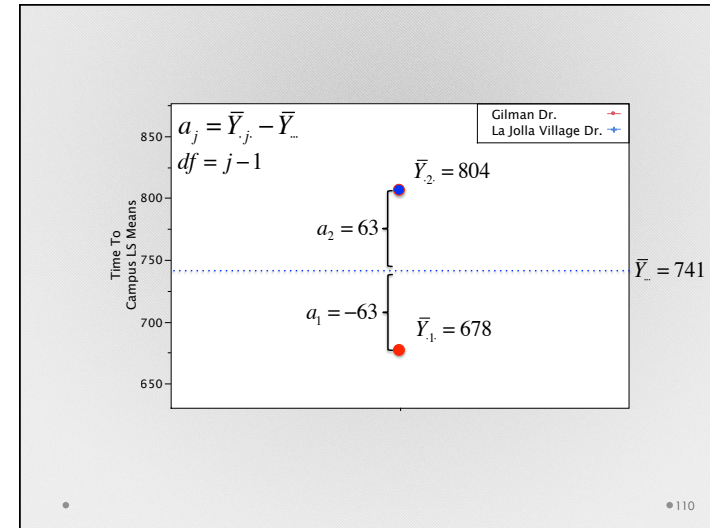
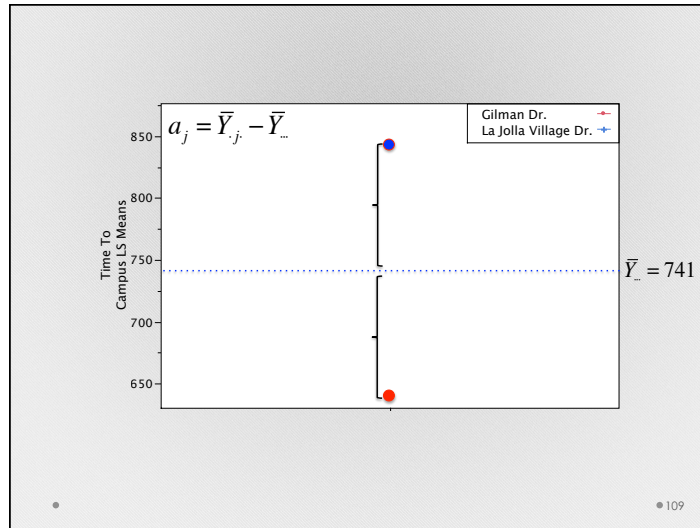
• 105

$$\bar{Y}_{jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | | $a_j = \bar{Y}_{.j} - \bar{Y}_{...}$ |
|----------|---------------------------------|------------------------------------|-------------------------------------|--------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $a_1 = -63$ $b_1 = 16$ | $a_1 = -63$ $b_2 = -16$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $a_2 = 63$ $b_1 = 16$ | $a_2 = 63$ $b_2 = -16$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

• 106

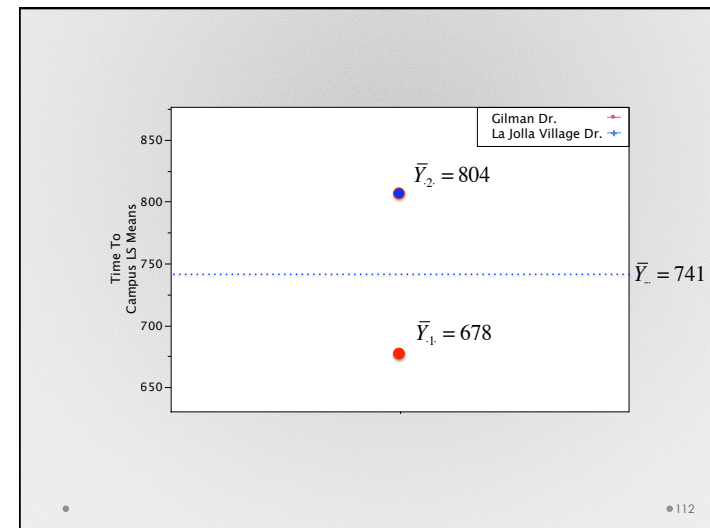


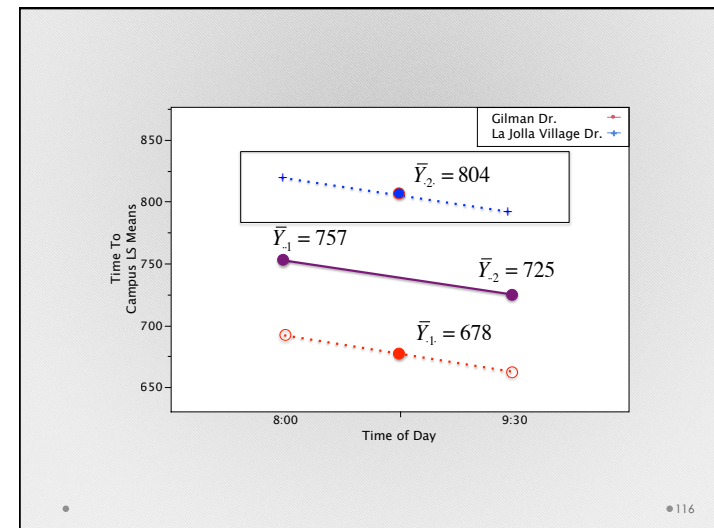
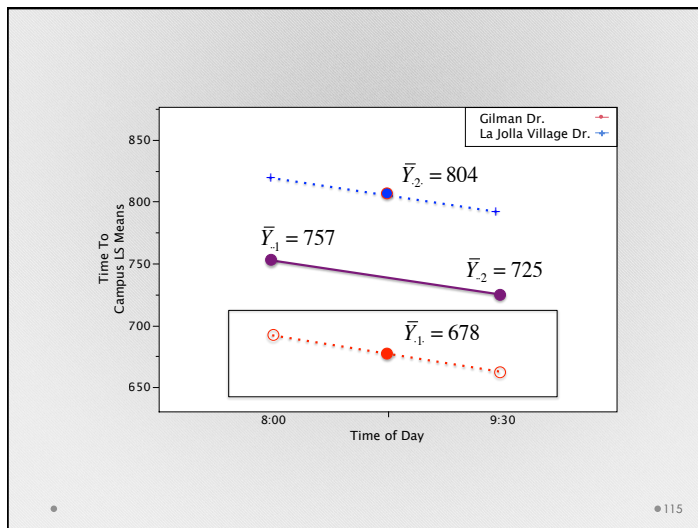
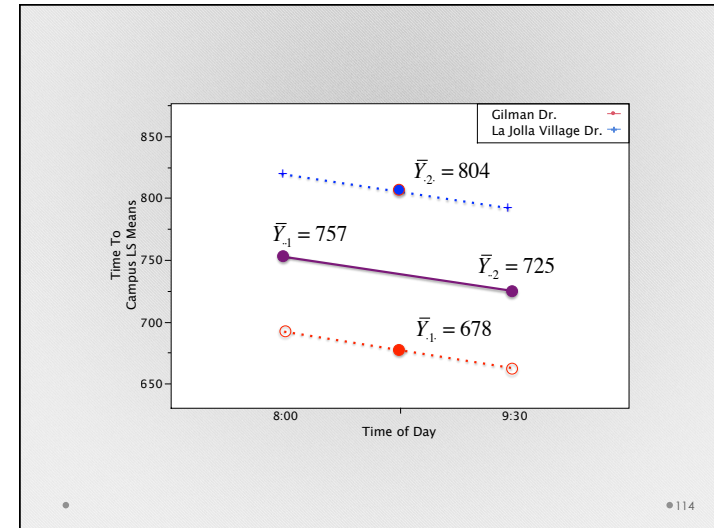
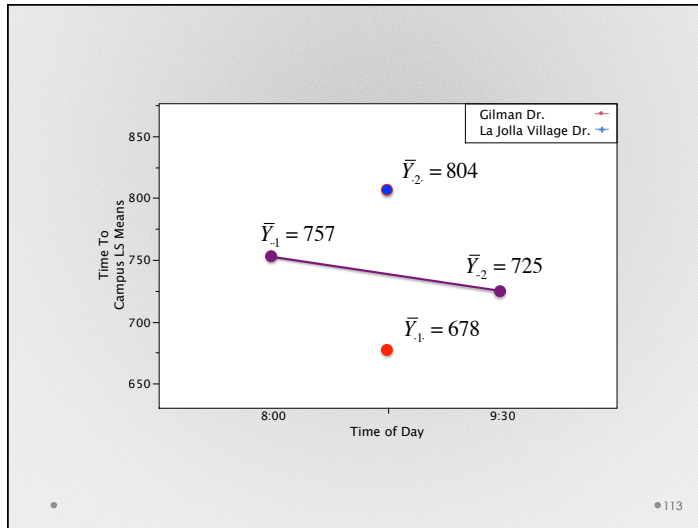


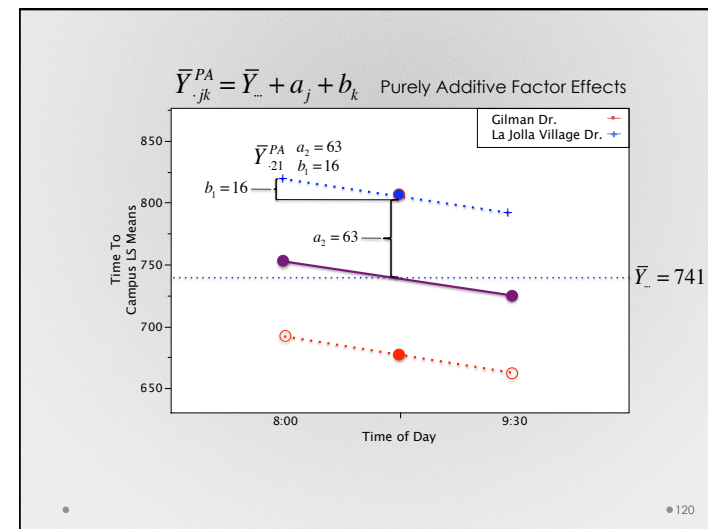
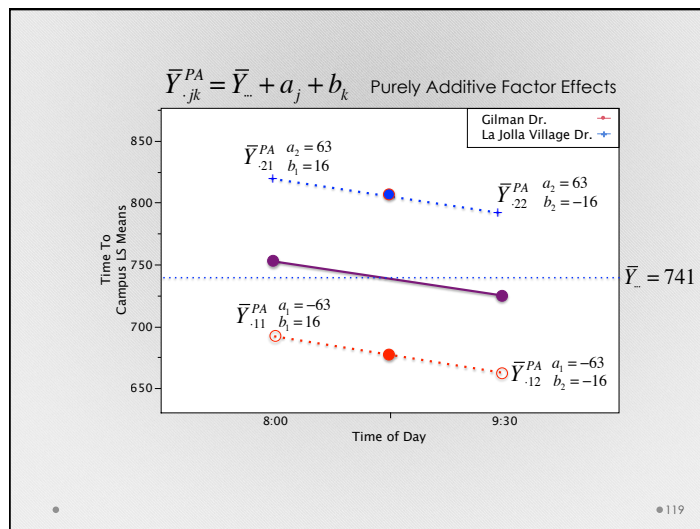
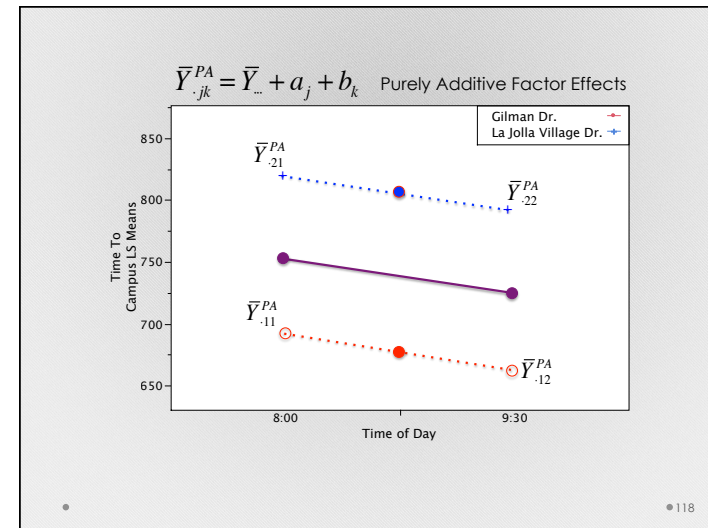
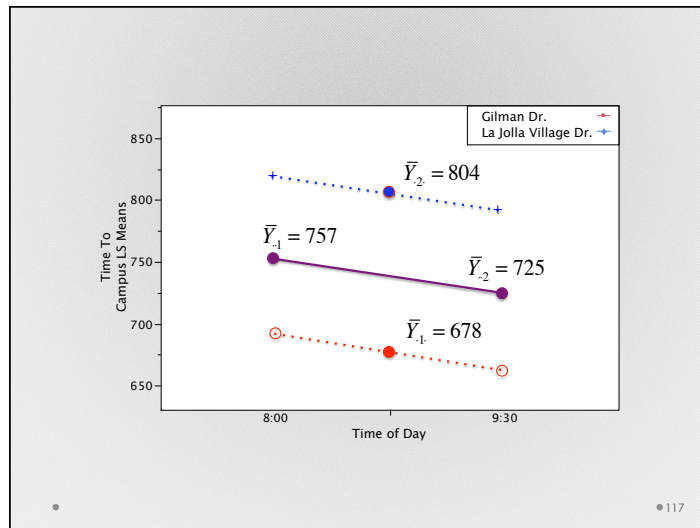
$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

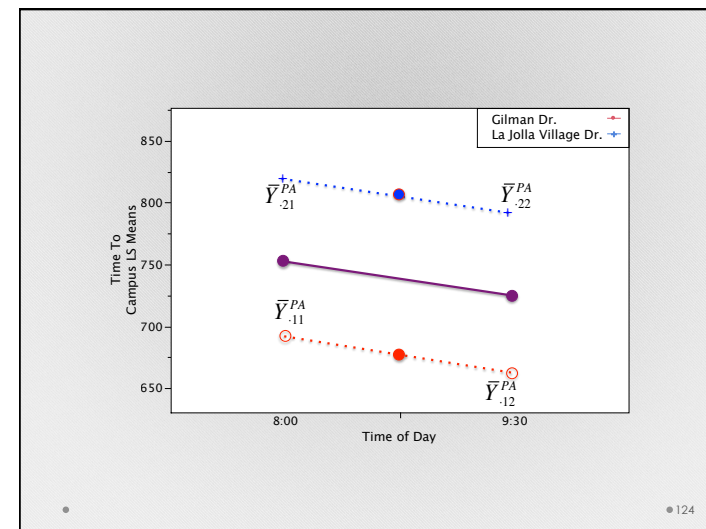
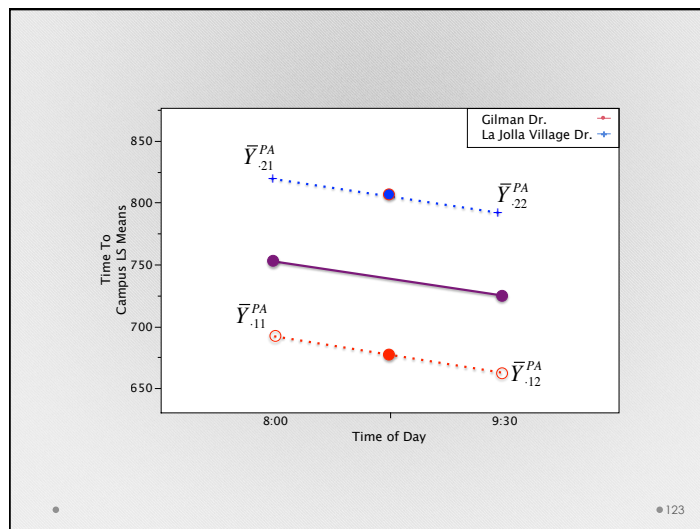
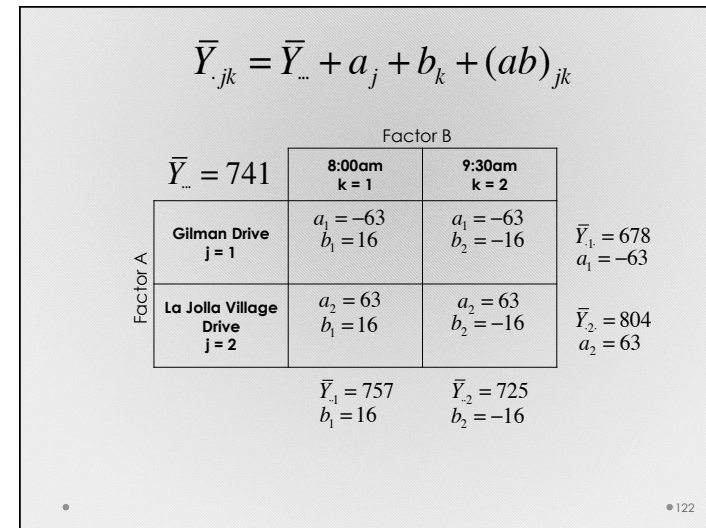
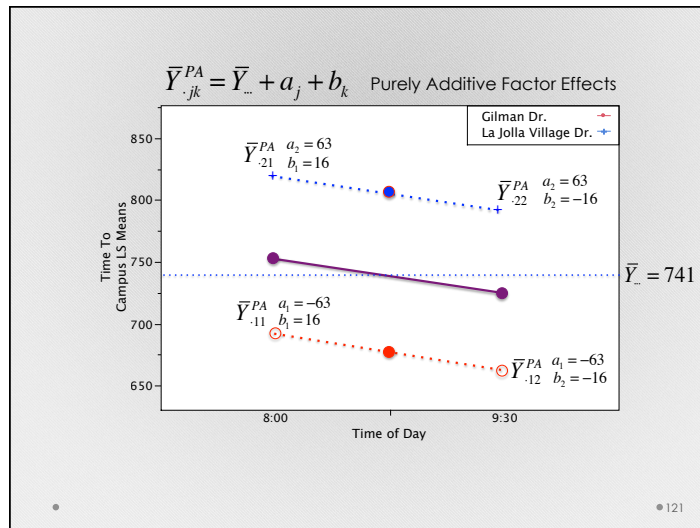
| | | Factor B | | |
|----------|--|------------------------------------|-------------------------------------|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | $\bar{Y}_{...} = 741$ Gilman Drive j = 1 | $a_1 = -63$ $b_1 = 16$ | $a_1 = -63$ $b_2 = -16$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $a_2 = 63$ $b_1 = 16$ | $a_2 = 63$ $b_2 = -16$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

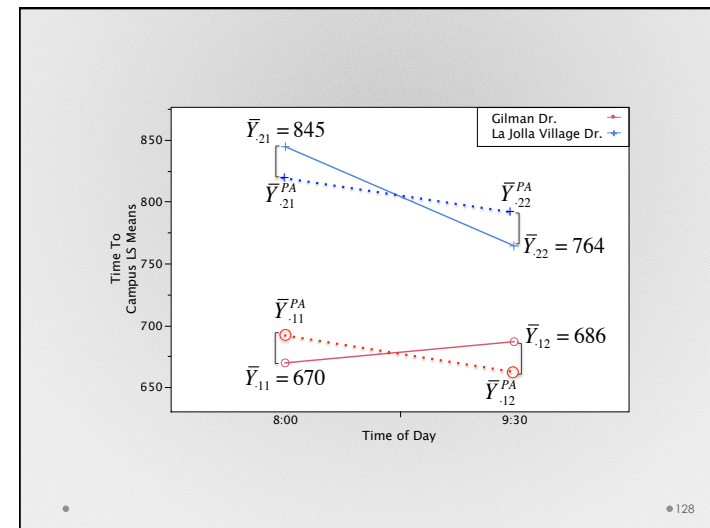
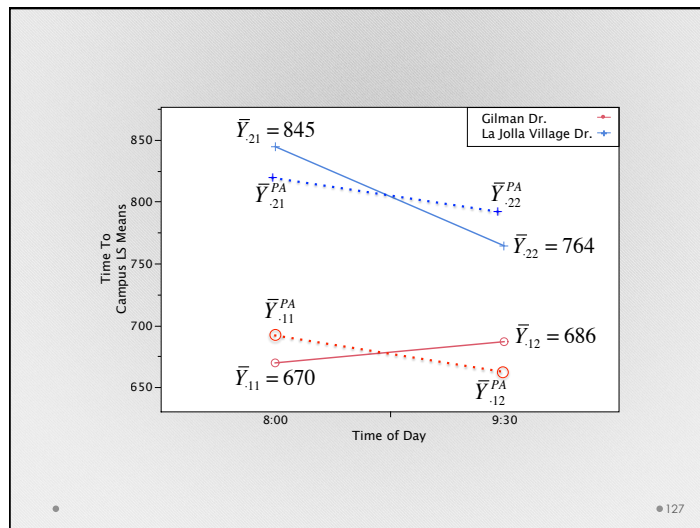
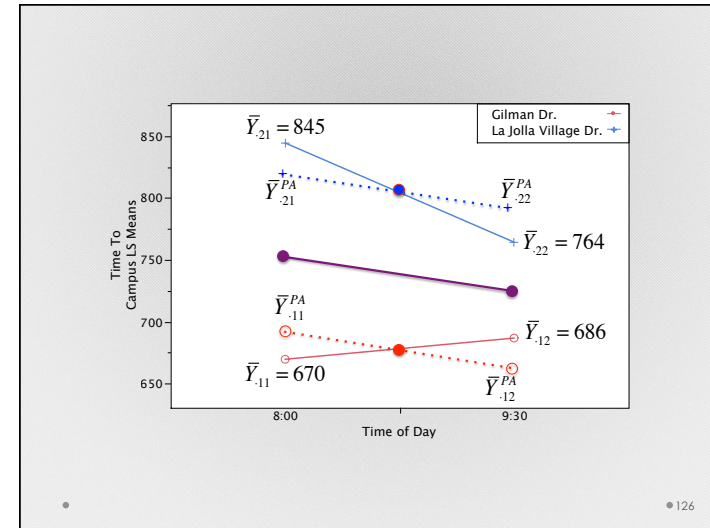
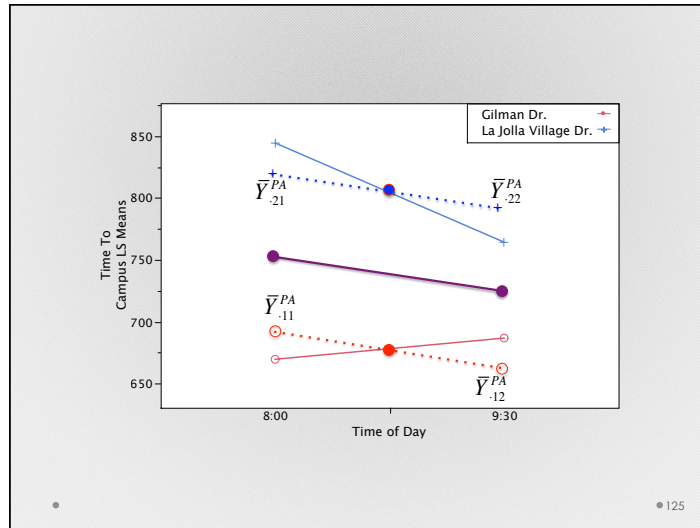
111







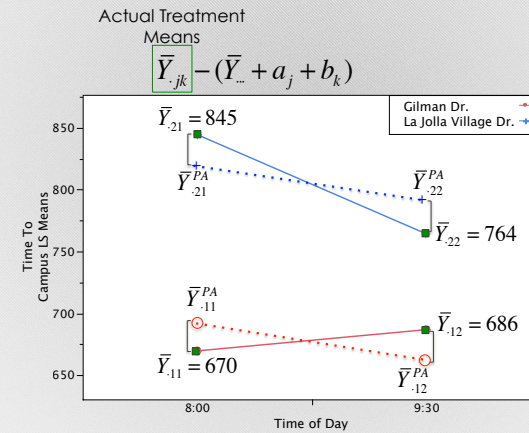
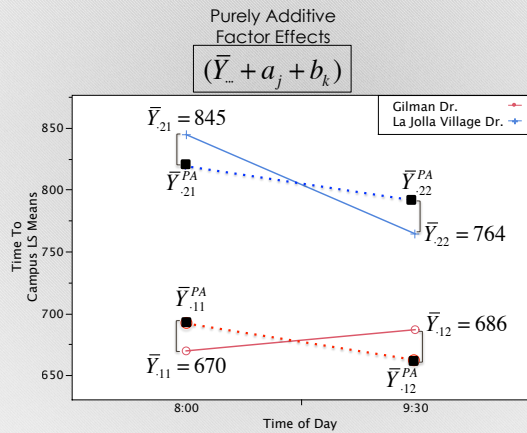
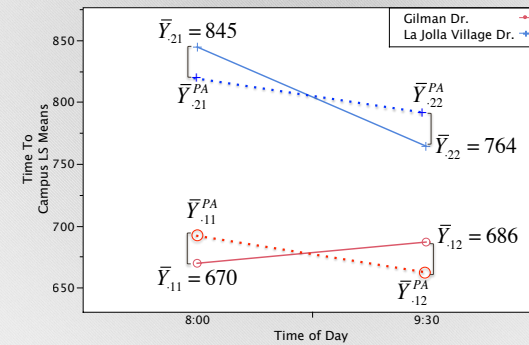


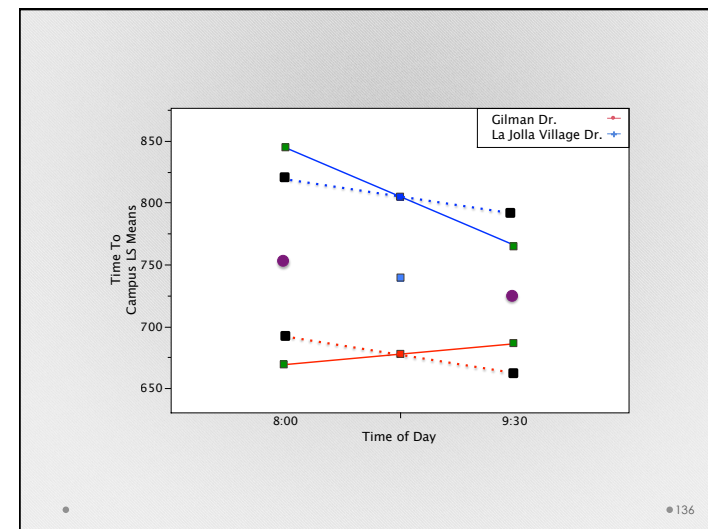
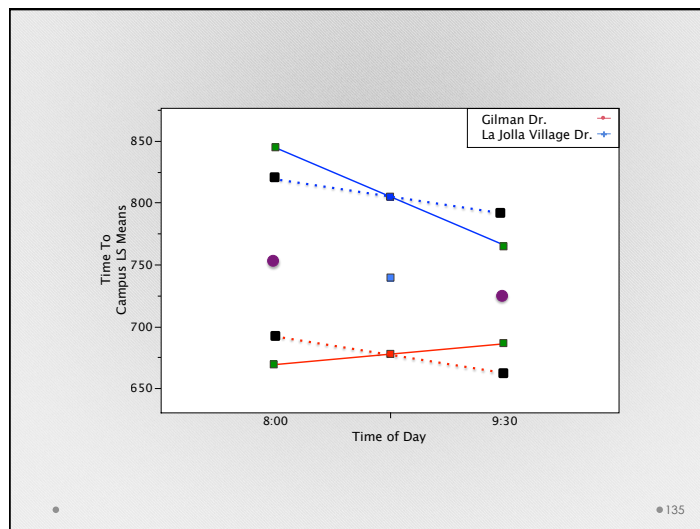
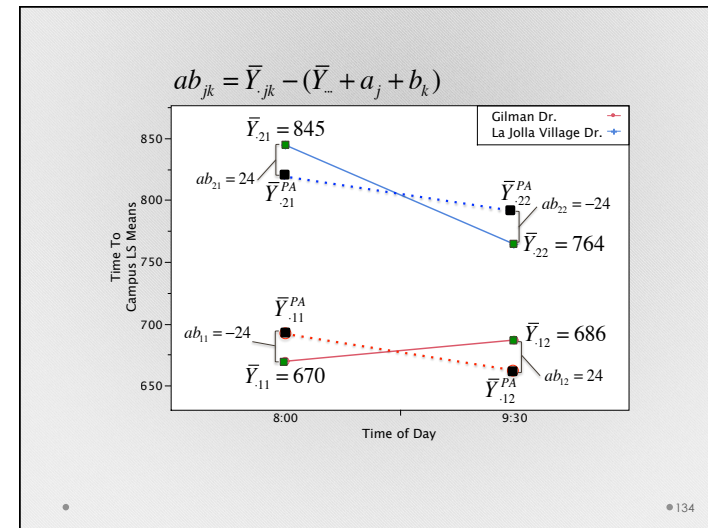
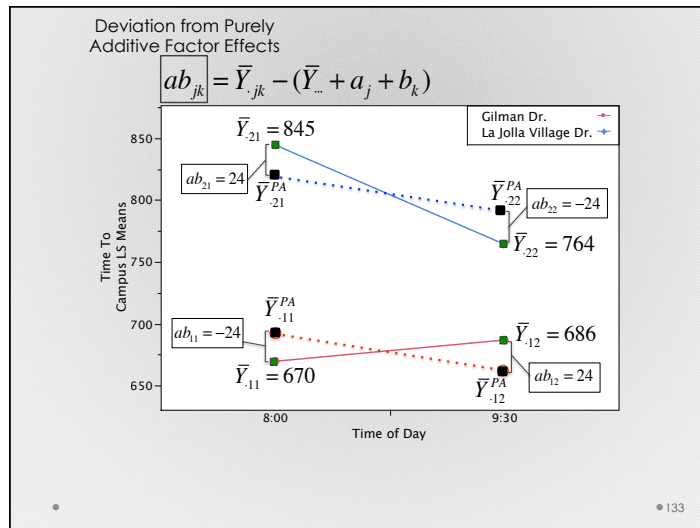


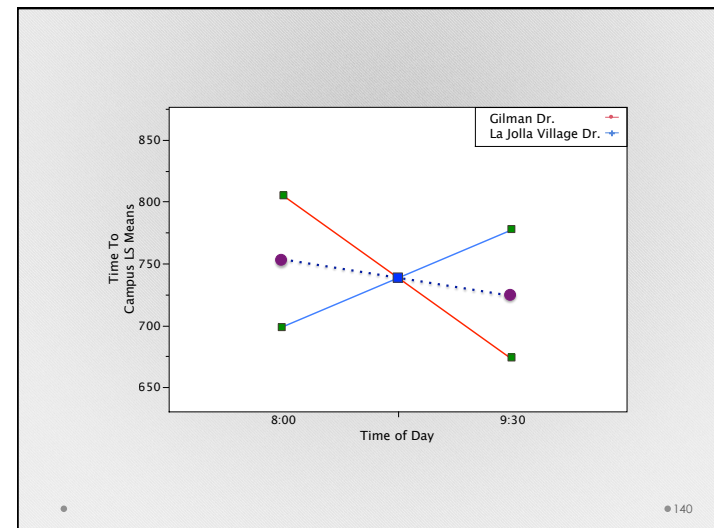
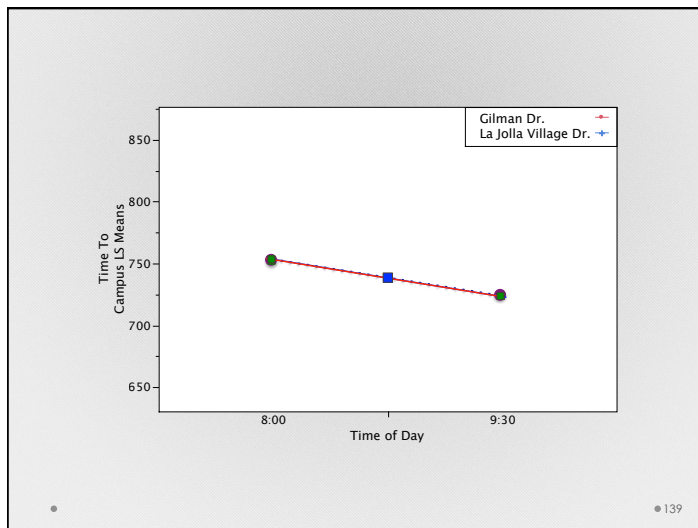
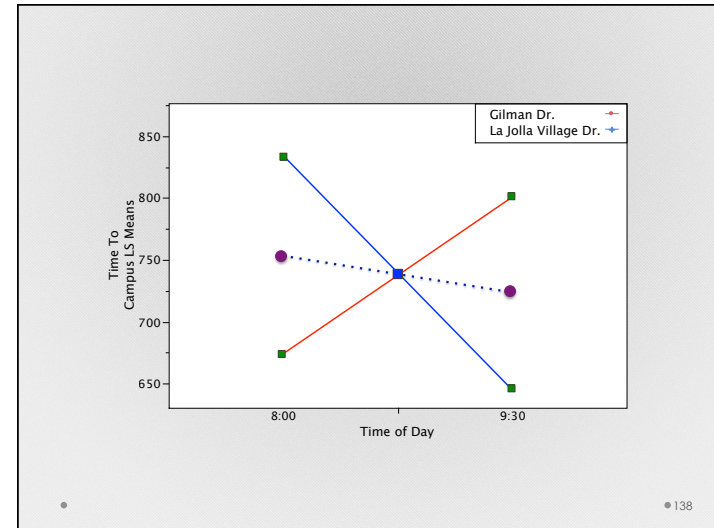
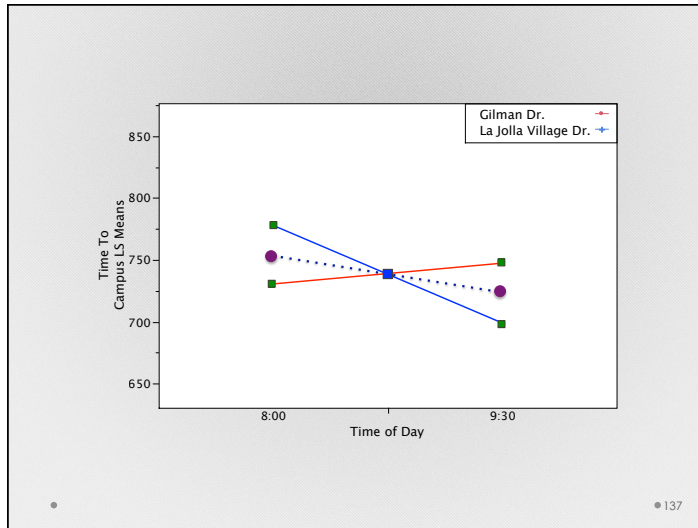
Interaction

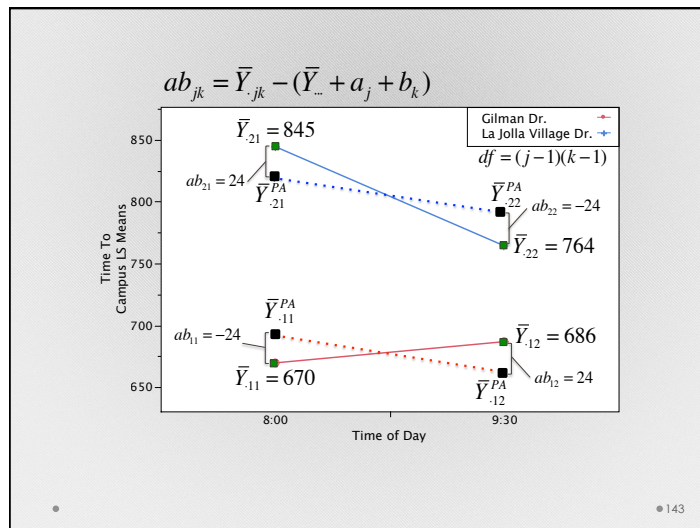
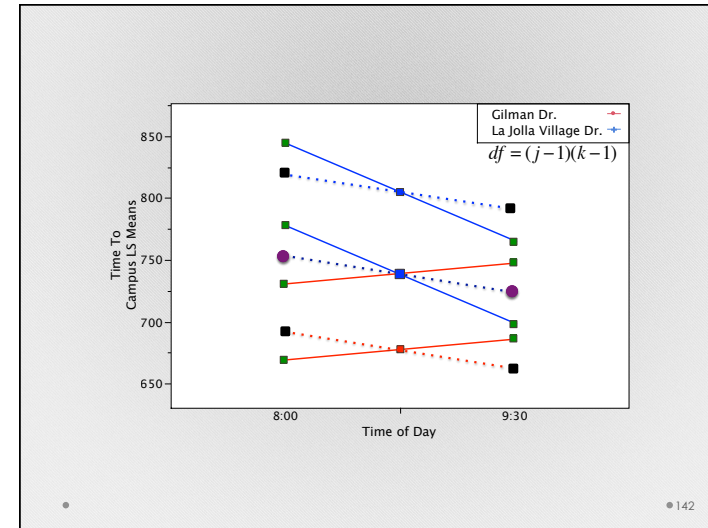
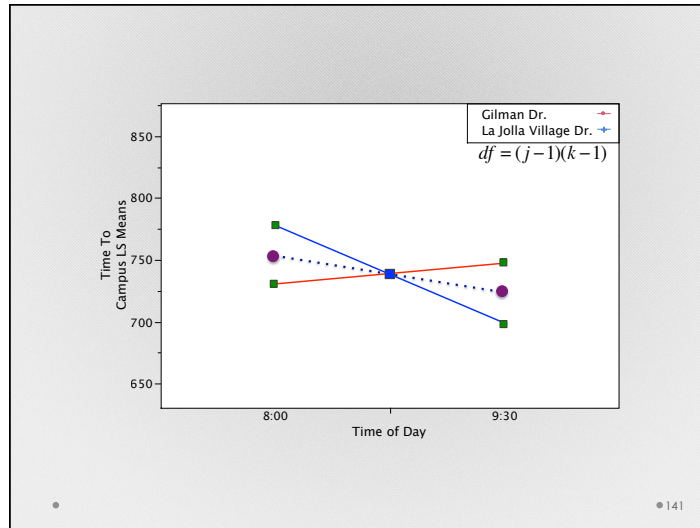
The interaction between factors is the degree to which the effects of one factor depend on the level of the other factor

The interaction between factors is the degree to which the mean for treatments differs from the additive effects of factors









$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | | |
|----------|------------------------------------|------------------------------------|-------------------------------------|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $a_1 = -63$ $b_1 = 16$ | $a_1 = -63$ $b_2 = -16$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $a_2 = 63$ $b_1 = 16$ | $a_2 = 63$ $b_2 = -16$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | | |
|----------|------------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | $\bar{Y}_{...} = 741$ | | | |
| | Gilman Drive j = 1 | $a_1 = -63$ $b_1 = 16$ $ab_{11} = -24$ | $a_1 = -63$ $b_2 = -16$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $a_2 = 63$ $b_1 = 16$ $ab_{21} = 24$ | $a_2 = 63$ $b_2 = -16$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

● 145

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | | |
|----------|------------------------------------|-----------------------|-----------------------|--|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | | | | |
| | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ | $\bar{Y}_{.12} = 686$ | |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ | $\bar{Y}_{.22} = 764$ | |

● 146

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | | |
|----------|------------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | $\bar{Y}_{...} = 741$ | | | |
| | Gilman Drive j = 1 | $a_1 = -63$ $b_1 = 16$ $ab_{11} = -24$ | $a_1 = -63$ $b_2 = -16$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $a_2 = 63$ $b_1 = 16$ $ab_{21} = 24$ | $a_2 = 63$ $b_2 = -16$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

● 147

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

| | | Factor B | | |
|----------|------------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | $\bar{Y}_{...} = 741$ | | | |
| | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

● 148

$$\bar{Y}_{.11} = \bar{Y}_{...} + a_1 + b_1 + (ab)_{11}$$

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | $\bar{Y}_{...} = 741$ | | | |
| | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{-1} = 757$ $b_1 = 16$ | $\bar{Y}_{-2} = 725$ $b_2 = -16$ | |

• 149

$$\bar{Y}_{.11} = \boxed{\bar{Y}_{...}} + a_1 + b_1 + (ab)_{11}$$

$$\bar{Y}_{.11} = 741 +$$

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | $\boxed{\bar{Y}_{...} = 741}$ | | | |
| | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{-1} = 757$ $b_1 = 16$ | $\bar{Y}_{-2} = 725$ $b_2 = -16$ | |

• 150

$$\bar{Y}_{.11} = \bar{Y}_{...} + \boxed{a_1} + b_1 + (ab)_{11}$$

$$\bar{Y}_{.11} = 741 + (-63) +$$

| | | Factor B | | |
|----------|---------------------------------|--|--|---|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | $\bar{Y}_{...} = 741$ | | | |
| | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $\boxed{a_1 = -63}$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{-1} = 757$ $b_1 = 16$ | $\bar{Y}_{-2} = 725$ $b_2 = -16$ | |

• 151

$$\bar{Y}_{.11} = \bar{Y}_{...} + a_1 + \boxed{b_1} + (ab)_{11}$$

$$\bar{Y}_{.11} = 741 + (-63) + (16) +$$

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | $\bar{Y}_{...} = 741$ | | | |
| | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{-1} = 757$ $\boxed{b_1 = 16}$ | $\bar{Y}_{-2} = 725$ $b_2 = -16$ | |

• 152

$$\bar{Y}_{.11} = \bar{Y}_{...} + a_1 + b_1 + (ab)_{11}$$

$$\bar{Y}_{.11} = 741 + (-63) + (16) + (-24) =$$

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

• 153

$$\bar{Y}_{.11} = \bar{Y}_{...} + a_1 + b_1 + (ab)_{11}$$

$$\bar{Y}_{.11} = 741 + (-63) + (16) + (-24) = 670$$

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

• 154

$$\bar{Y}_{.21} = \bar{Y}_{...} + a_2 + b_1 + (ab)_{21}$$

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

• 155

$$\bar{Y}_{.21} = \bar{Y}_{...} + a_2 + b_1 + (ab)_{21}$$

$$\bar{Y}_{.11} = 741 +$$

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

• 156

$$\bar{Y}_{.21} = \bar{Y}_{...} + a_2 + b_1 + (ab)_{21}$$

$$\bar{Y}_{.11} = 741 + (63) +$$

Factor B

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

• 157

$$\bar{Y}_{.21} = \bar{Y}_{...} + a_2 + b_1 + (ab)_{21}$$

$$\bar{Y}_{.11} = 741 + (63) + (16) +$$

Factor B

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

• 158

$$\bar{Y}_{.21} = \bar{Y}_{...} + a_2 + b_1 + (ab)_{21}$$

$$\bar{Y}_{.11} = 741 + (63) + (16) + (24) =$$

Factor B

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

• 159

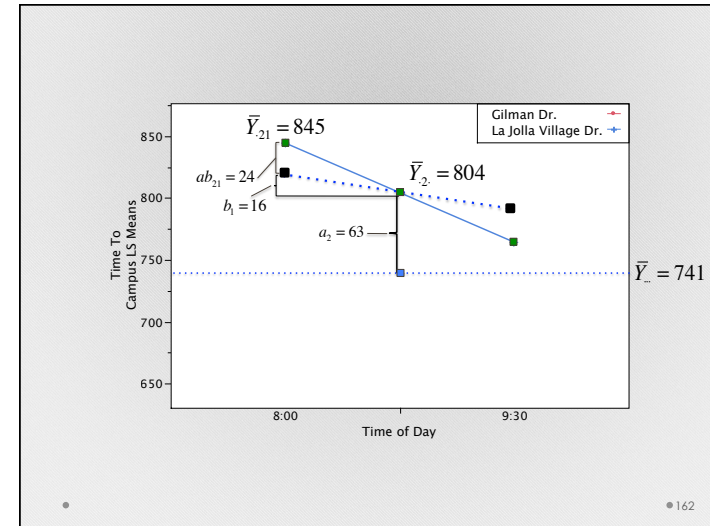
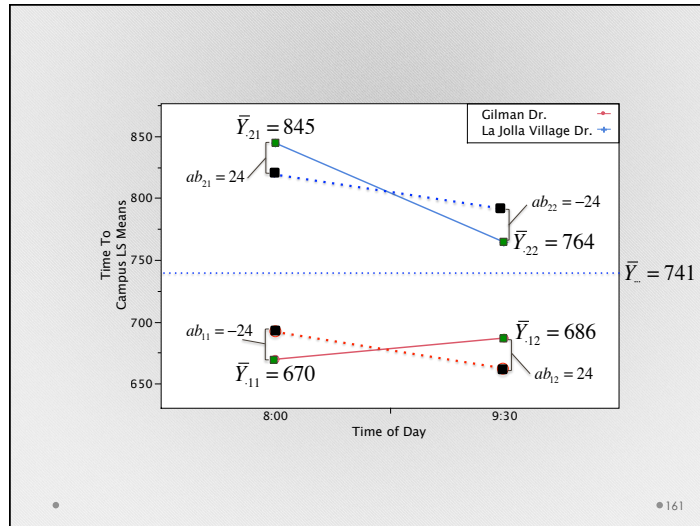
$$\bar{Y}_{.21} = \bar{Y}_{...} + a_2 + b_1 + (ab)_{21}$$

$$\bar{Y}_{.11} = 741 + (63) + (16) + (24) = 845$$

Factor B

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

• 160



$$\bar{Y}_{.11} = \bar{Y}_{...} + a_1 + b_1 + (ab)_{11}$$

$$\bar{Y}_{.11} = 741 + (63) + (16) + (24) = 845$$

Factor B

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

Factor B

| | | Factor B | | |
|----------|---------------------------------|--|--|-------------------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 | |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ $ab_{11} = -24$ | $\bar{Y}_{.12} = 686$ $ab_{12} = 24$ | $\bar{Y}_{.1} = 678$ $a_1 = -63$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ $ab_{21} = 24$ | $\bar{Y}_{.22} = 764$ $ab_{22} = -24$ | $\bar{Y}_{.2} = 804$ $a_2 = 63$ |
| | | $\bar{Y}_{.1} = 757$ $b_1 = 16$ | $\bar{Y}_{.2} = 725$ $b_2 = -16$ | |

Two Factor Linear Model
(Sample Model for Treatment Mean)

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

Mean of Y for the jth treatment of factor A, and kth treatment of factor B = Grand Mean + Effect offset for level j of Factor A + Effect offset for level k of Factor B + Effect offset for unique effect of Factors in treatment jk

• 165

Two Factor Linear Model
(Sample Model for Treatment Mean)

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

Mean of Y for the jth treatment of factor A, and kth treatment of factor B = Grand Mean + Effect offset for level j of Factor A + Effect offset for level k of Factor B + Effect offset for unique effect of Factors in treatment jk

• 166

Two Factor Linear Model
(Sample Model for Treatment Mean)

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

Mean of Y for the jth treatment of factor A, and kth treatment of factor B = Grand Mean + Effect offset for level j of Factor A + Effect offset for level k of Factor B + Deviation from perfectly additive effects for treatment jk

• 167

Two Factor Linear Model
(Sample Model for Treatment Mean)

$$\bar{Y}_{.jk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk}$$

Mean of Y for the jth treatment of factor A, and kth treatment of factor B = Grand Mean + Effect offset for level j of Factor A + Effect offset for level k of Factor B + Deviation from perfectly additive effects for treatment jk

• 168

Two Factor Linear Model
(Sample Model)

$$Y_{ijk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk} + e_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Deviation from perfectly additive effects for treatment *jk* + Error

● 169

Two Factor Linear Model
(Sample Model)

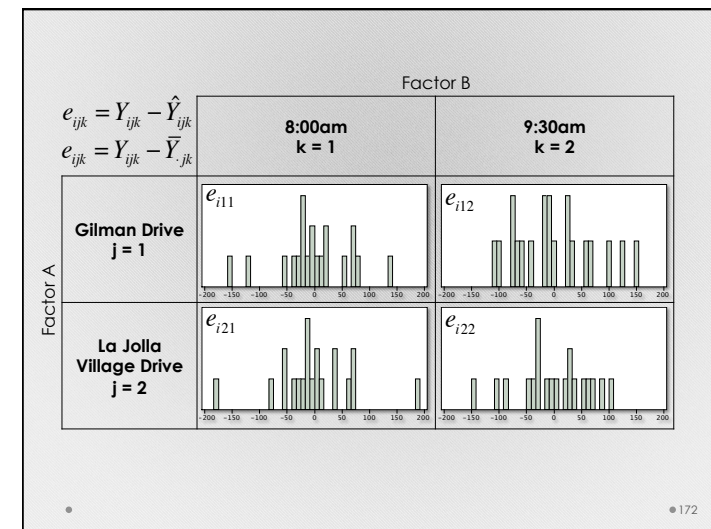
$$Y_{ijk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk} + e_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Deviation from perfectly additive effects for treatment *jk* + Error

● 170

| | | Factor B | |
|----------|---------------------------------|-----------------------|-----------------------|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | Gilman Drive j = 1 | $\bar{Y}_{.11} = 670$ | $\bar{Y}_{.12} = 686$ |
| | La Jolla Village Drive j = 2 | $\bar{Y}_{.21} = 845$ | $\bar{Y}_{.22} = 764$ |

● 171



Two Factor Linear Model (Sample Model)

$$Y_{ijk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk} + e_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Deviation from perfectly additive effects for treatment *jk* + Error

• 173

Two Factor Linear Model (Population Model)

$$Y_{ijk} = \mu_{...} + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

Score on Y for the *i*th individual in the *j*th treatment of factor A, and *k*th treatment of factor B = Grand Mean + Effect offset for level *j* of Factor A + Effect offset for level *k* of Factor B + Deviation from perfectly additive effects for treatment *jk* + Error

• 174

| | | Factor B | |
|----------|------------------------|-------------|-------------|
| | | 8:00am | 9:30am |
| Factor A | Gilman Drive | 670 Seconds | 686 Seconds |
| | La Jolla Village Drive | 845 Seconds | 764 Seconds |

Possible Tests:

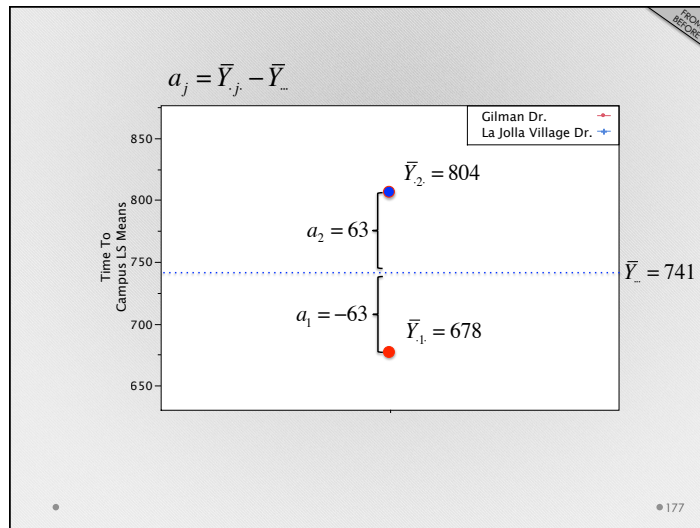
- Overall Effect of Route (ignoring time)
- Overall Effect of Time (ignoring route)
- Effects of Specific Route & Time Combinations
(degree to which effect of one factor depends on the level of the other factor)

• 175

Tests of Effects for Two Factors

- Overall Effect of Factor A
 - Main Effect of A
 - Test of whether all $\alpha_j = 0$

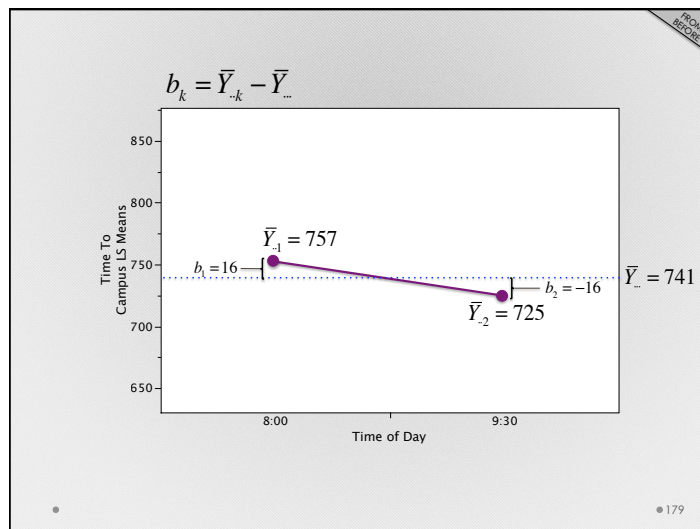
• 176



Tests of Effects for Two Factors

- Overall Effect of Factor A
 - Main Effect of A
 - Test of whether all $\alpha_i = 0$
- Overall Effect of Factor B
 - Main Effect of B
 - Test of whether all $\beta_k = 0$

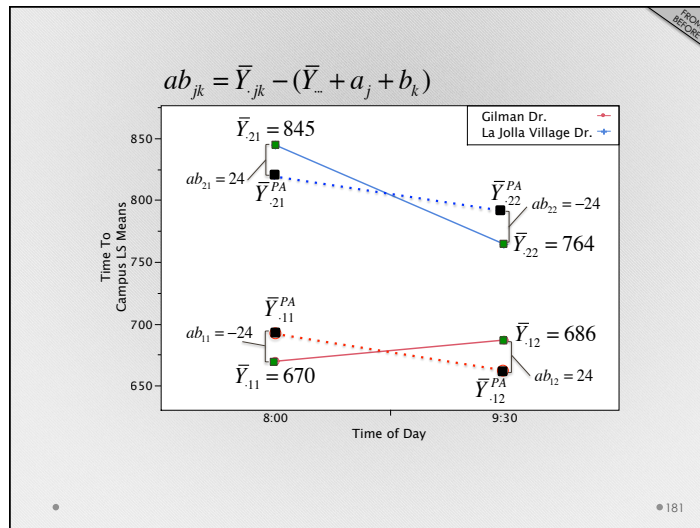
178



Tests of Effects for Two Factors

- Overall Effect of Factor A
 - Main Effect of A
 - Test of whether all $\alpha_i = 0$
- Overall Effect of Factor B
 - Main Effect of B
 - Test of whether all $\beta_k = 0$
- Effects of Specific A & B Combinations:
 - Interaction between A and B
 - Test of whether all $(\alpha\beta)_{jk} = 0$

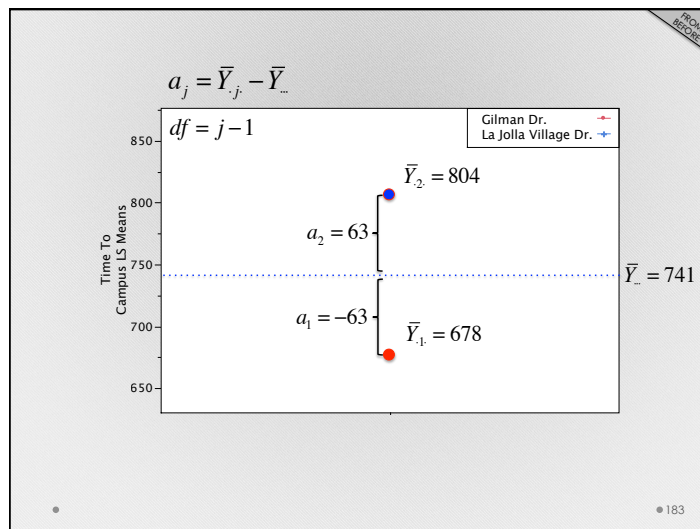
180



Tests of Effects for Two Factors

- Overall Effect of Factor A
 - Main Effect of A
 - Test of whether all $\alpha_i = 0$
- Effects of Specific A & B Combinations:
 - Interaction between A and B
 - Test of whether all $(\alpha\beta)_{jk} = 0$
- Overall Effect of Factor B
 - Main Effect of B
 - Test of whether all $\beta_k = 0$

182



Tests of Effects for Two Factors

- Overall Effect of Factor A
 - Main Effect of A
 - Test of whether all $\alpha_i = 0$
- Effects of Specific A & B Combinations:
 - Interaction between A and B
 - Test of whether all $(\alpha\beta)_{jk} = 0$
- Overall Effect of Factor B
 - Main Effect of B
 - Test of whether all $\beta_k = 0$

184

Tests of Effects for Two Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

- Overall Effect of Factor B
 - Main Effect of B
 - Test of whether all $\beta_k = 0$

- Effects of Specific A & B Combinations:
 - Interaction between A and B
 - Test of whether all $(\alpha\beta)_{jk} = 0$

• 185

Tests of Effects for Two Factors

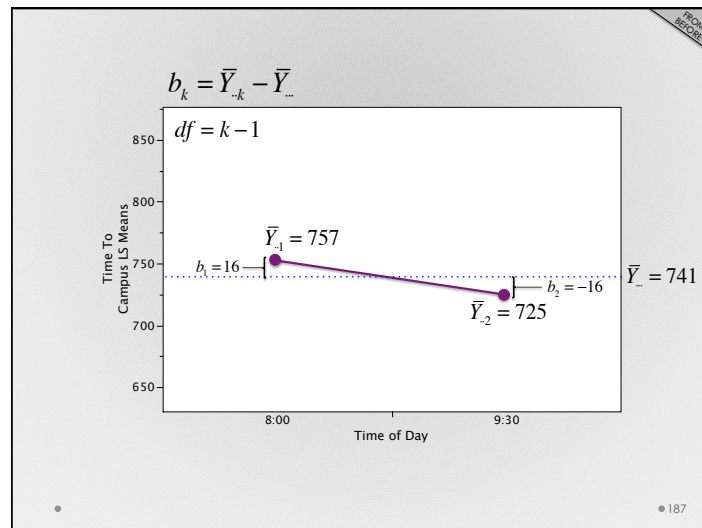
- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

- Overall Effect of Factor B
 - Main Effect of B
 - Test of whether all $\beta_k = 0$

- Effects of Specific A & B Combinations:
 - Interaction between A and B
 - Test of whether all $(\alpha\beta)_{jk} = 0$

• 186



Tests of Effects for Two Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

- Overall Effect of Factor B
 - Main Effect of B
 - Test of whether all $\beta_k = 0$

- Effects of Specific A & B Combinations:
 - Interaction between A and B
 - Test of whether all $(\alpha\beta)_{jk} = 0$

• 188

Tests of Effects for Two Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

- Overall Effect of Factor B

$$F_B = \frac{MS_B}{MS_{error}}$$

- Effects of Specific A & B Combinations:

- Interaction between A and B
- Test of whether all $(\alpha\beta)_{jk} = 0$

• 189

Tests of Effects for Two Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

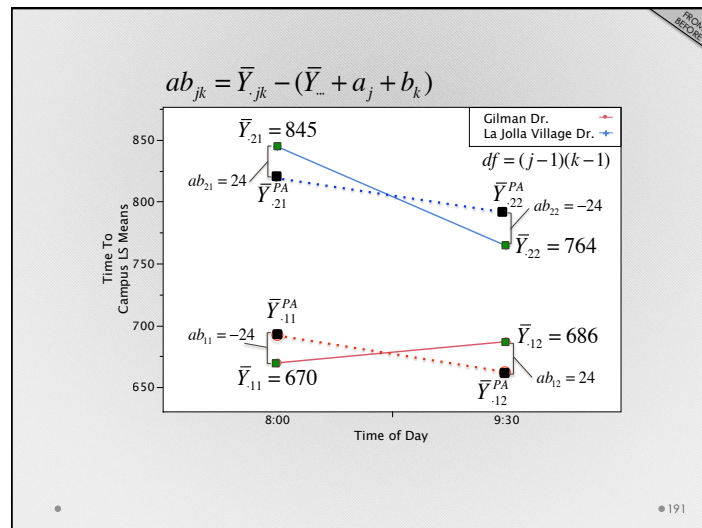
- Overall Effect of Factor B

$$F_B = \frac{MS_B}{MS_{error}}$$

- Effects of Specific A & B Combinations:

- Interaction between A and B
- Test of whether all $(\alpha\beta)_{jk} = 0$

• 190



Tests of Effects for Two Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

- Overall Effect of Factor B

$$F_B = \frac{MS_B}{MS_{error}}$$

- Effects of Specific A & B Combinations:

- Interaction between A and B
- Test of whether all $(\alpha\beta)_{jk} = 0$

• 192

Tests of Effects for Two Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

- Overall Effect of Factor B

$$F_B = \frac{MS_B}{MS_{error}}$$

- Effects of Specific A & B Combinations:

$$F_{AB} = \frac{MS_{AB}}{MS_{error}}$$

• 193

Tests of Effects for Two Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

- Overall Effect of Factor B

$$F_B = \frac{MS_B}{MS_{error}}$$

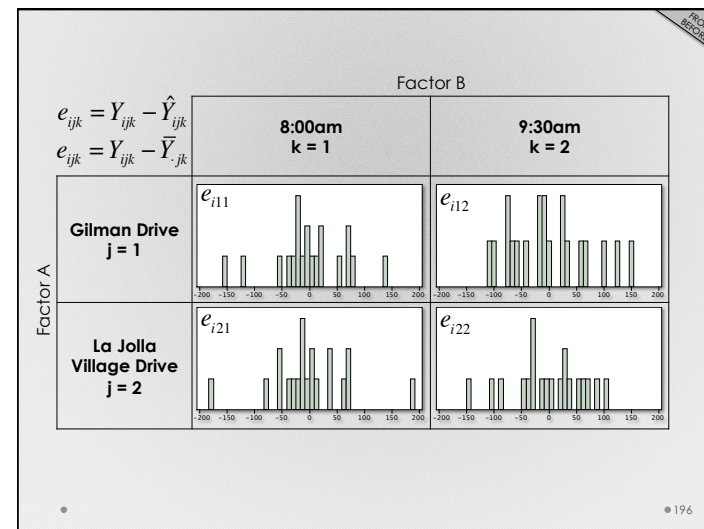
- Effects of Specific A & B Combinations:

$$F_{AB} = \frac{MS_{AB}}{MS_{error}}$$

• 194

| | | Factor B | |
|------------------------------------|---|----------------------------|----------------------------|
| | | 8:00am k = 1 | 9:30am k = 2 |
| Factor A | $e_{ijk} = Y_{ijk} - \hat{Y}_{ijk}$ $e_{ijk} = Y_{ijk} - \bar{Y}_{\cdot jk}$ | | |
| | | | |
| Gilman Drive j = 1 | | $\bar{Y}_{\cdot 11} = 670$ | $\bar{Y}_{\cdot 12} = 686$ |
| La Jolla Village Drive j = 2 | | $\bar{Y}_{\cdot 21} = 845$ | $\bar{Y}_{\cdot 22} = 764$ |

• 195



• 196

Two Factor Linear Model
(Sample Model)

$$Y_{ijk} = \bar{Y}_{...} + a_j + b_k + (ab)_{jk} + e_{ijk}$$

Score on Y
for the ith
individual in
the jth
treatment of
factor A, and
kth treatment
of factor B

=

Grand
Mean

+

Effect
offset for
level j of
Factor A

+

Effect
offset for
level k of
Factor B

+

Deviation
from
perfectly
additive
effects for
treatment
jk

+

Error

● 197

Tests of Effects for Two Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$
- Overall Effect of Factor B

$$F_B = \frac{MS_B}{MS_{error}}$$

• Effects of Specific A & B
Combinations:

$$F_{AB} = \frac{MS_{AB}}{MS_{error}}$$

● 198